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Module
6

Applied Mathematics 20

CIRCLES

Learning Technologies Branch

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LEARNING

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Module

6

Applied Mathematics 20

CIRCLES

Learning Technologies Branch

Alberta

LEARNING

Applied Mathematics 20
Module 6: Circles
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-1947-0

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Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

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- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

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Welcome

Applied Mathematics 20

Module 1
Graphs

Module 2
Non-Linear Functions

Module 3
Linear Systems

Module 4
Linear Programming

Module 5
Finance

Module 6
Circles

Module 7
Measurement

Welcome to Module 6. We hope you'll enjoy your study of Circles.

Applied Mathematics 20 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

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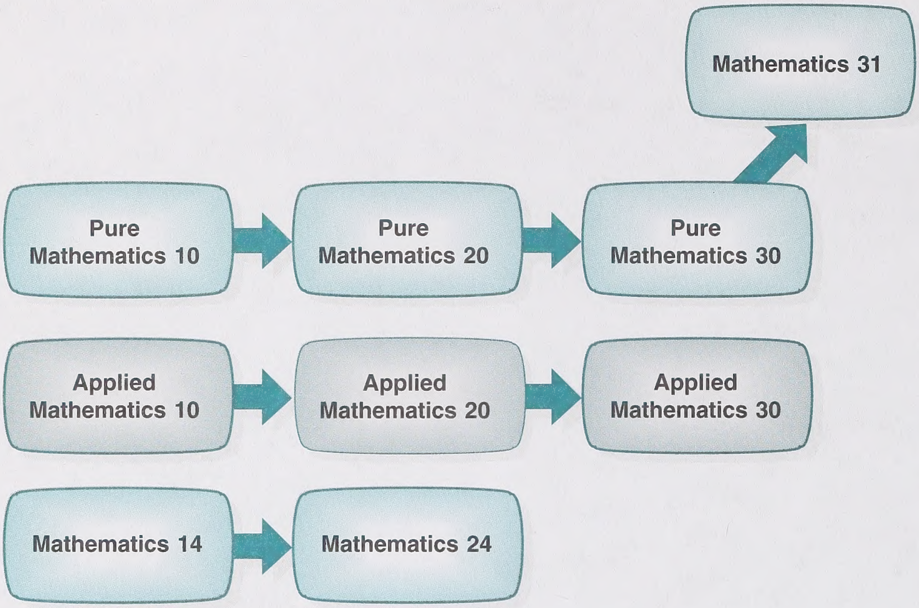
Introduction to Applied Mathematics 20

Applied Mathematics 20 is the second course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read “Questions and Answers About Senior High School Mathematics” and “The New Senior High School Mathematics Program and Post-Secondary Studies.” These documents can be found at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 20, it is recommended that you talk with a school counsellor about your career plans.

TRANSFERRING FROM THE APPLIED PROGRAM

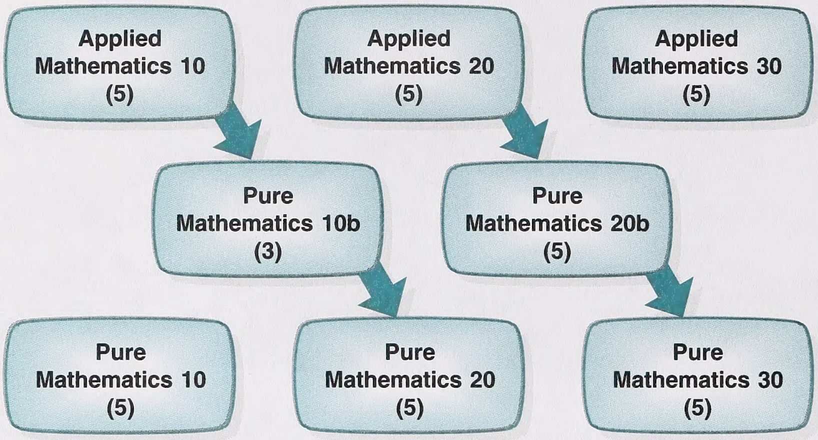
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won’t have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

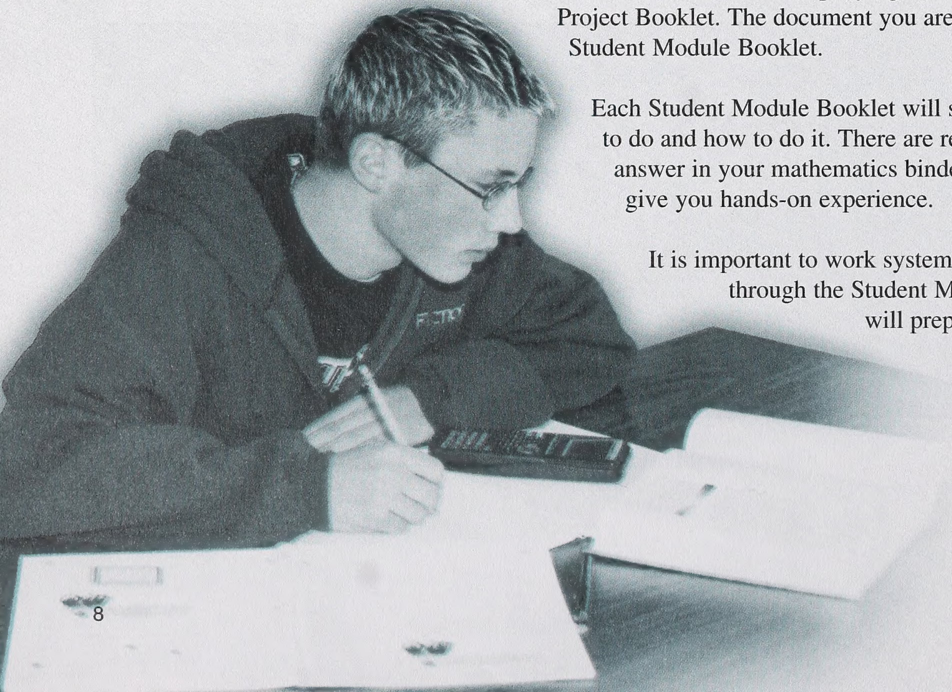


STRATEGIES FOR COMPLETING APPLIED MATHEMATICS 20

For each module in Applied Mathematics 20, there is a Student Module Booklet, an accompanying Assignment Booklet, and a Project Booklet. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words.

  Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.

Resources You Will Need

In addition to the distance learning materials for Applied Mathematics 20, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 11 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2000)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel.*

- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-82**	EL-9600C	Algebra FX 2.0	HP 39g
TI-83	EL-9600*	CFX-9850 GA-Plus*	
TI-83 Plus	EL-9200*	CFX-9850 G*	
TI-86	EL-9300*	CFX-9800 G*	
TI-89		FX-9700 series*	
TI-92*			
TI-92 Plus			

*no longer commercially available

**The TI-82 calculator will remain on the approved list for the 2000–2001 and 2001–2002 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor*.

Many of the resources that you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

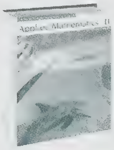
http://www.lrc.learning.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

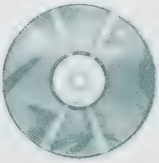
Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

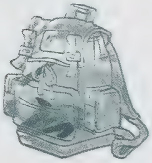
- Refer to the textbook.



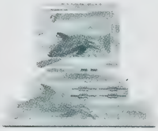
- Refer to the Applied Mathematics 20 CD.



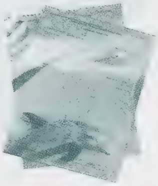
- Use mathematical instruments or measuring tools.



- Work with a computer.



- Complete the module project or assignment.



- Explore the Internet.

Remember: Any Internet website address given in this module is subject to change.

MODULE OVERVIEW

Have you ever noticed how often circles pop up in everyday places? There are the buttons on clothes, the wheels on vehicles, the clocks on most school walls, and the shapes of toys (such as balls and tops) to name a few.

In games and leisure activities, like the game of baseball, circles and other interesting geometrical shapes appear in many places. Often, the properties of circles are used without even realizing it. Can you think of a good reason as to why most buttons are round? How many automobile companies use the circle as the basis for their logos? Do you see all the circles around you, or are they so common that they don't even catch your attention?

In this module, you will be working with technology in the form of an interactive geometry program. This technology will assist you as you look for properties of a circle and of angles and lines formed in and around circles. You will first study the program itself; then you will use it to explore interesting and useful facts about circle geometry. You will then focus on angles formed on arcs of a circle, chords joining points on a circle, quadrilaterals with all vertices on a circle, and the measure of interior angles of polygons.

ASSESSMENT

Accompanying this Student Module Booklet is a Project Booklet and an Assignment Booklet. Your grading in this module will be based upon the module project and the module assignment you submit for evaluation. The mark distribution is as follows:

Module Project	**40 marks**
Module Assignment	**60 marks**
TOTAL	**100 marks**

Remember that Activities 1 to 5 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your solutions with the complete solutions provided in the Appendix. If your answer does not agree with the one given, you can learn where the error is by carefully following the solution in the Appendix.

MODULE PROJECT

CIRCLE DESIGN

Beginning the Project

> *And since geometry is the right foundation of all painting, I have decided to teach its rudiments and principles to all youngsters eager for art…*
>
> **—Albrecht Dürer (1471–1528)**

Circles are a central theme for many groups of Aboriginal peoples. Examples of Aboriginal designs that incorporate the circle and some of its properties are medicine wheels, dream catchers, as well as various art designs. A medicine wheel is made of stones arranged in a circle with additional stones leading to the centre, like spokes on a wheel. Medicine wheels are places for healing, teaching, and understanding. A dream catcher is a web-like design constructed on a ring. The web forms a circular hole at the centre. According to Aboriginal traditions, the web will catch your good ideas and dreams and allow the bad ones to go through the hole.

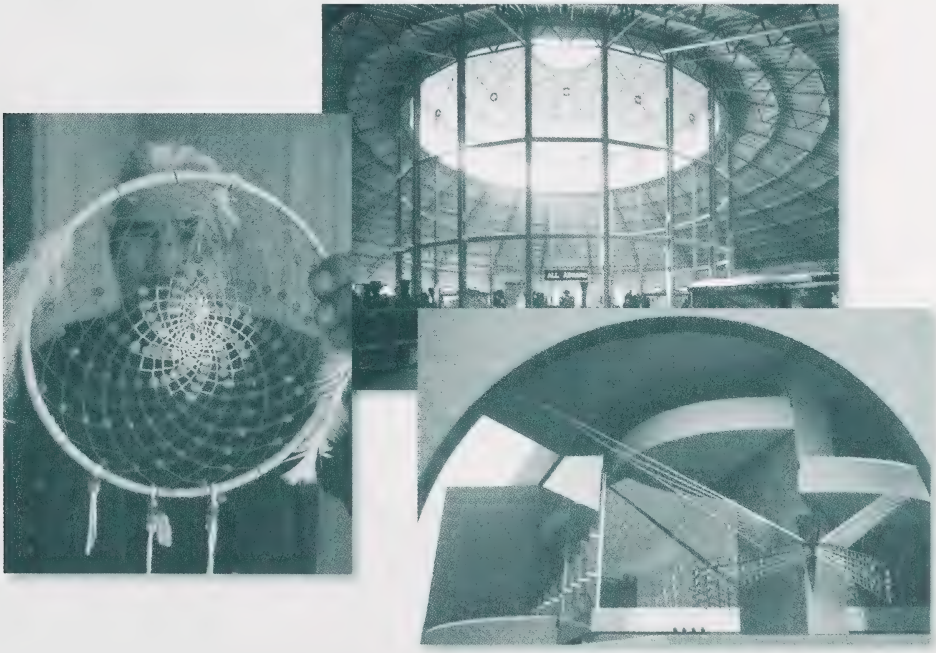
In your project for Module 6: Circles, you are going to use the circle and many of its properties as the basis of an interesting project that you will create and design. You can view many Aboriginal circle designs on the Internet by entering words like *medicine wheel* and *dream catcher* into one of the Internet's search engines.

Turn to page 292 of the textbook and read "Circle Design." Once you have read the page, complete the questions posed. Store your responses in the project section of your mathematics binder.

After you have recorded some of your initial ideas for your circle design, begin researching some of the things you will need to understand in order to build your design. In this module you will be working with properties of circles. You should try to incorporate as many of these properties as possible to your design. You may wish to begin your research by visiting the Internet site described on page 293 of the textbook. Once there, you can read about some of the things others have done with circles in their designs. Be sure to keep notes of your findings in your journal. You should also keep track of the websites you visit and other materials that you use for research as these will be needed for the report you will be writing about your artwork. You may wish to bookmark the best Internet sites on your computer so you can revisit them easily.

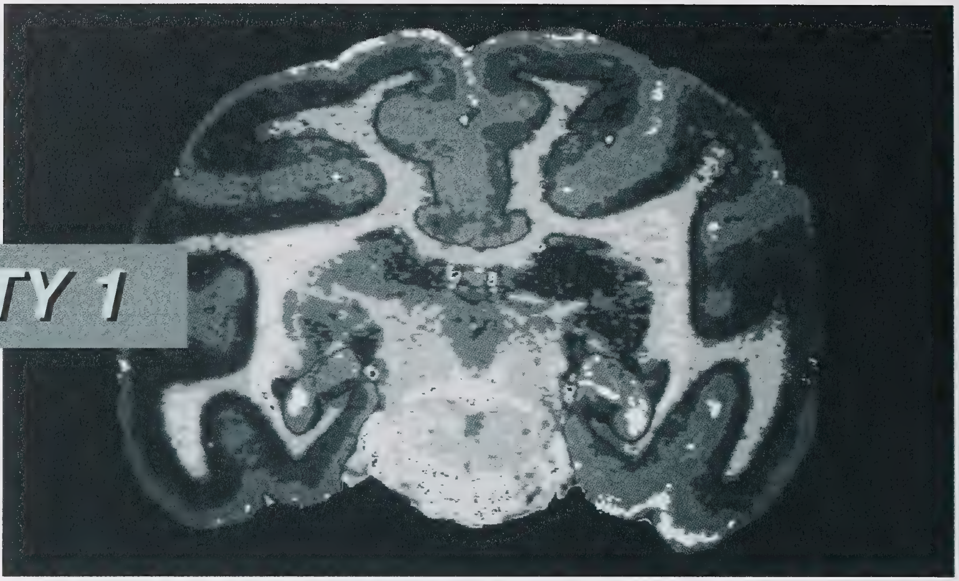
As you work through Activities 1 to 5, continue researching ideas on the kind of design you want to make and what medium you want to make it with. You can do research on the Internet, in magazines, in books, and in nature. You may find talking to artists and designers useful as well.

The following shows examples of circles used in designs. Some of the ideas may be useful to you as you work on your ideas for a circle design.



You will be given more direction on how to complete this project later in the module. In the meantime, feel free to discuss this project with a study partner or a family member. Remember, the work on the project that you submit must be your own.

ACTIVITY 1



Using Interactive Geometry Software

I cannot do it without comp[u]ters.
—William Shakespeare (1564–1616)

Computers can make many tasks much easier. In fact, like the computer-assisted scan shown, they make things that could only be dreamed about in your grandparents' youth. Computer technology has revolutionized society. It has certainly changed the way we think about and work with mathematics. Interactive geometry software is a tool that simplifies the task of examining geometric truths.

In this activity you will discover how to use interactive geometry software, like *The Geometer's Sketchpad*. In the other activities you will use this software to test hypotheses and construct geometric examples.

You may wish to check if you can access *The Geometer's Sketchpad* through your school. It is available for purchase through the Learning Resource Centre (LRC) as well as software distribution outlets.

If you do not have access to a computer with *The Geometer's Sketchpad*, there are alternative activities for you to complete. If you do not have *The Geometer's Sketchpad* but have access to another, similar program, such as *Cabri Geometry II*, you will have to read the software's user's guide to learn the software. Many geometry software products will allow you to carry out the needed activities; they just function a bit differently.

You may wish to search the Internet for programs that you would like to use. The following Internet site has a list of interactive geometry software that you might find useful:

http://mathforum.com/dynamic/classroom.html

If you are using interactive geometry software, do exercise 1. If you are not using interactive geometry software, do exercise 2.

1. Turn to pages 294 and 295 of the textbook and work through exercises 1 and 2 of "Investigation 1: The Toolbox." **Note:** This investigation is specific to *The Geometer's Sketchpad* (as are all activities from the textbook). If you are using a different program, complete the suggested activities using your software.

2. Use your compass to draw several **circles**; then use your ruler to draw a **radius** for each circle. Label the centre of each circle and the point where the radius intersects the circle. Place three or more additional points on each circle and create a cyclic polygon by joining each point to the next point along the circumference of the circle.

Compare your responses with the suggested answers in the Appendix, Activity 1, page 55.

When you have completed Investigation 1, you should be able to use your software to create a circle, a **line segment**, and a **point**. You should also be able to label the items you create.

If you are using interactive geometry software, do exercise 3. If you are not using interactive geometry software, do exercise 4.

3. Turn to pages 296 to 298 of the textbook and work through exercises 1 to 3 of "Investigation 2: The Measure Menu." If you are using software other than *The Geometer's Sketchpad*, complete the suggested activities using your software.

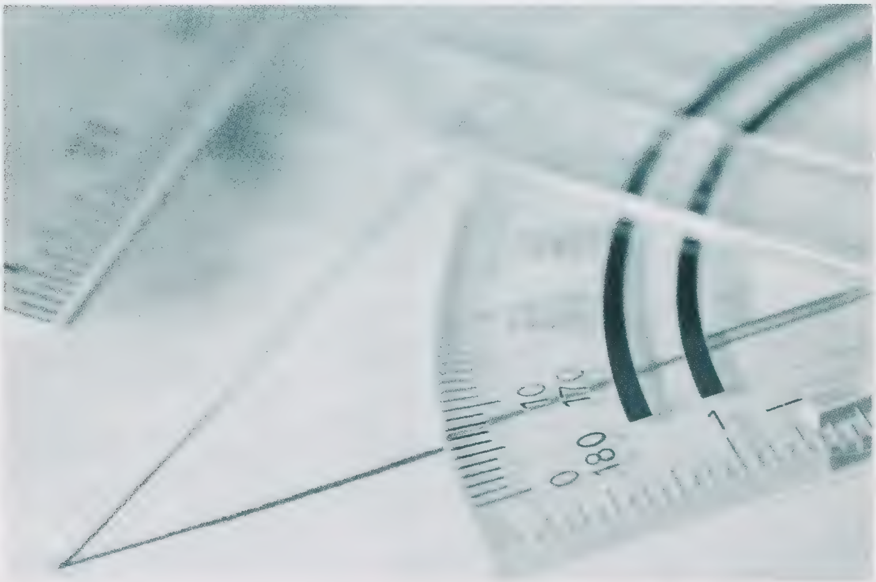
To clear the screen from the previous investigation, do the following:

- Click on the Selection arrow.
- Click on "Select All" from the Edit menu.
- Click on "Clear" from the Edit menu.

Note: You can also select "New Sketch" from the File menu.

4. Use your protractor to measure the interior angles in your cyclic polygons drawn earlier, and use your ruler to measure the radius of each of your circles. Find the area of each of your circles using the formula $A = \pi r^2$.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 55–56.



When you have completed Investigation 2, you will be able to use your methods to create circles and polygons and determine the measures of the areas and lengths of the items you create.

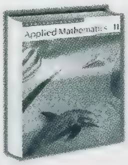
Turn to page 298 of the textbook and study “Sketchpad Tips” closely. These tips will help you avoid a number of possible problems while using *The Geometer’s Sketchpad*.

5. If you are using interactive software, turn to page 299 of the textbook and complete exercises 1 to 4 of “Discussing the Ideas.”

Compare your responses with the suggested answers in the Appendix, Activity 1, page 57.

LOOKING BACK

In this activity you discovered that a dynamic software, like *The Geometer’s Sketchpad*, can be used to create geometric diagrams and explore measurements and relationships in these diagrams.



6. Turn to page 299 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 1, page 57.

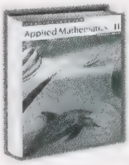
ACTIVITY 2

Properties of Chords in Circles

> *We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others.*
>
> **—Blaise Pascal (1623 –1662)**

Imagine a group, in the early days, out on a hunting expedition. As night falls, they start a fire and prepare for the night. The moon is shining brightly in the sky and everything seems as it always has been. Then the moon starts to disappear. They see a black, circular shadow creep over the moon, covering it in darkness. This is not what the hunters would expect and it would certainly make a major impression on them. The large, circular shadow of Earth covering the circular moon, inching over it slowly, gives some very interesting shapes. (You may even want to consider some of them for your module project.)

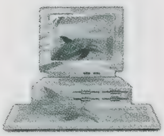
The mathematics of circles intersecting circles can be very complex. In this activity you will work with a slightly simpler topic. You will study **chords of a circle** and their properties.



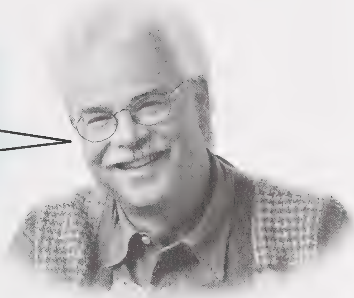
Turn to page 300 of the textbook and read the introduction to Tutorial 6.2, "Properties of Chords in Circles."

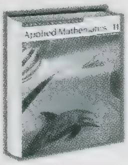
1. Answer exercises 1 and 2 of "Practise Your Prior Skills" on page 300 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 2, page 58.



Now, you will investigate some properties of chords of a circle. If you are using interactive geometry software, do exercise 2.
If you do not have access to interactive geometry software, do exercise 3.





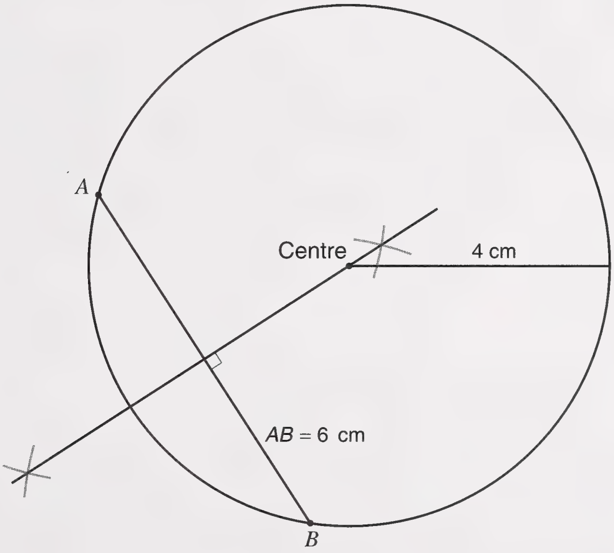
2. Turn to pages 301 and 302 of the textbook and do the following.

a. Complete Steps 1 to 3 of "Investigation 1: Chord Property 1," answering exercise 6.

b. Complete Step 4 of "Investigation 1: Chord Property 1," answering exercises 13 and 14.

c. Complete Step 5 of "Investigation 1: Chord Property 1," answering exercise 17.

3. Draw three circles with radii of 4 cm, 5 cm, and 6 cm. In each circle, draw a chord that is 6 cm in length; then construct a **perpendicular** from the centre of the circle to the chord (as shown in the following diagram).



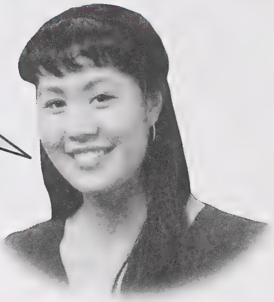
a. Measure the line segments created by the perpendiculars to the chords. Examine your constructions and the measures of the chord, including the two segments created by the perpendicular from the centre.

b. If you were to construct another circle with a chord and a perpendicular from the centre, what would you expect to be true of the lengths of the two segments of the chord?

c. Write a conjecture about the line that passes through the centre of a circle and is perpendicular to a chord.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 59–60.

The following exercises investigate the properties of the perpendicular bisector of a chord. If you are using interactive geometry software, do exercise 4. If you are not using interactive geometry software, do exercise 5.

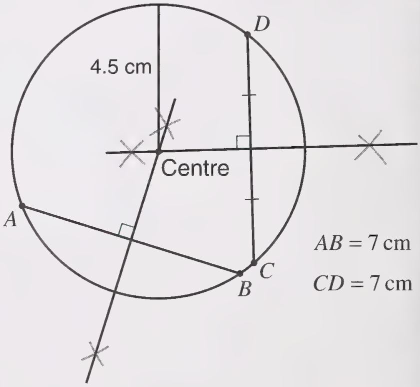


4. Turn to pages 302 and 303 of the textbook and do the following.

a. Complete Steps 1 to 3 of "Investigation 2: Chord Property 2," answering exercises 7 and 8.

b. Complete Step 4 of "Investigation 2: Chord Property 2," answering exercise 11.

5. Draw three circles with radii of 4.5 cm, 6 cm, and 7.5 cm. In each circle, draw two different chords 7 cm in length. For each chord, construct a perpendicular bisector of the chord (as shown in the diagram on the right).



a. Examine your constructions, paying particular attention to the perpendicular bisectors of the chords. What do you notice about the perpendicular bisector of a chord?

b. Write a conjecture about where the perpendicular bisectors of two chords in any circle will meet.

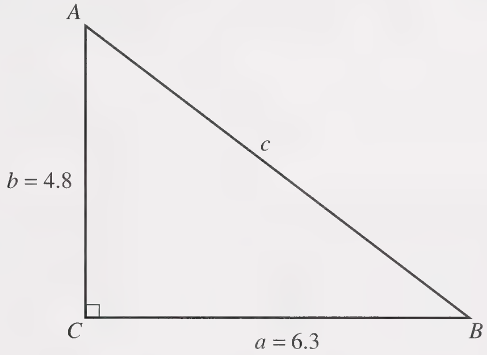
Compare your responses with the suggested answers in the Appendix, Activity 2, page 60.

Compare your conjectures regarding chords and perpendicular bisectors with "Properties of Chords" on page 303 of the textbook.

Both of the properties of chords you have worked with involve right angles (perpendicular lines). For the remainder of this module, you will have to watch closely for right triangles in order to solve the questions posed. In previous mathematics courses, you worked with a famous theorem for right-angled triangles, called the Pythagorean Theorem. Here is an example to help refresh your memory.

Example

Calculate the length of the hypotenuse, *c*, of the following right triangle. Round your answer to 2 decimal places.



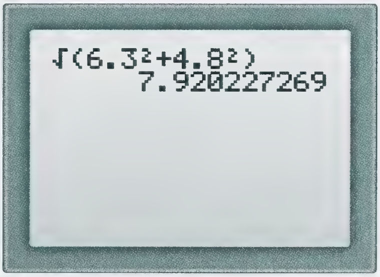
Solution

The Pythagorean Theorem states that the square of the length of the hypotenuse, *c*, is equal to the sum of the squares of the lengths of the other two sides, *a* and *b*. This can be expressed as follows:

$$c^2 = a^2 + b^2$$

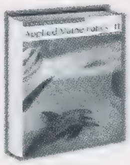
According to the diagram, $a = 6.3$ and $b = 4.8$. Substitute these values into the Pythagorean Theorem and simplify.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 6.3^2 + 4.8^2 \\ c^2 &= 39.69 + 23.04 \\ c^2 &= 62.73 \\ c &= \sqrt{62.73} \\ c &\doteq 7.92 \end{aligned}$$



```
√(6.3²+4.8²)
         7.920227269
```

The length of the hypotenuse is approximately 7.92.



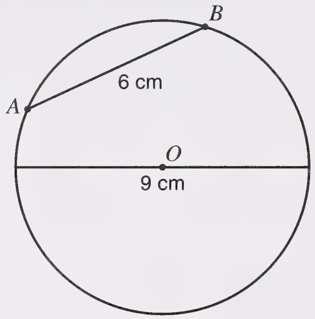
6. Turn to page 303 of the textbook and complete exercises 1.a. and 1.c. of “Practise Your Prior Skills.” Round your answer to 2 decimal places.

Compare your responses with the suggested answers in the Appendix, Activity 2, page 61.

Turn to page 304 of the textbook and work through “Example 1: Determine the distance from the centre of a circle to a chord.” Then work through the additional example that follows.

Example

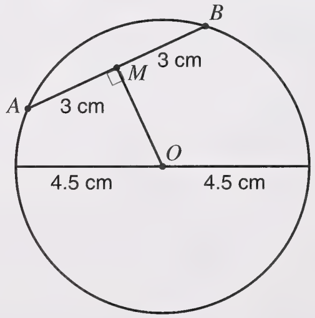
Determine the distance from chord *AB* to the centre of the circle, *O*, in the following diagram. Round your answer to 2 decimal places.



Solution

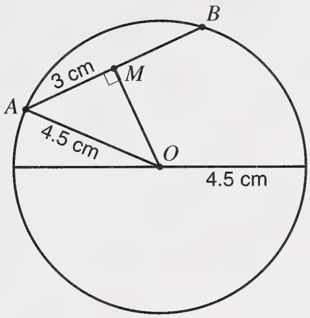
The diagram shows a diameter of a circle with a length of 9 cm. Therefore, the radius is half, or 4.5 cm.

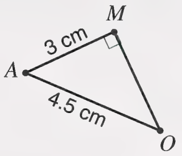
Draw a segment, *OM*, perpendicular to chord *AB*. The diagram now looks like the following.



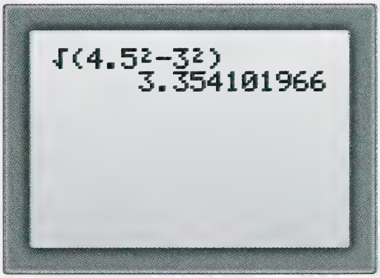
Segments *AM* and *MB* are each 3 cm in length, because a perpendicular from the centre to a chord bisects the chord.

Draw the radius *OA*. Now the diagram looks like the following, showing how the Pythagorean Theorem can be applied.





$$
\begin{aligned}
\therefore\ (OA)^2 &= (AM)^2 + (OM)^2 \\
(4.5)^2 &= (3.0)^2 + (OM)^2 \\
(OM)^2 &= (4.5)^2 - (3.0)^2 \\
(OM)^2 &= 20.25 - 9.00 \\
OM &= \sqrt{11.25} \\
&\doteq 3.35
\end{aligned}
$$

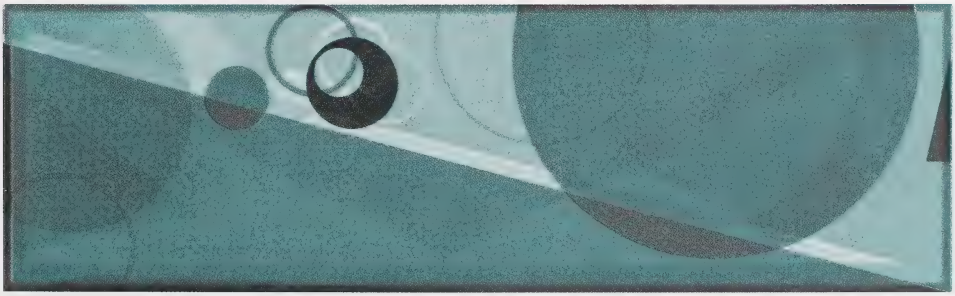


```
√(4.5²-3²)
        3.354101966
```

The distance of chord *AB* from the centre of the circle is approximately 3.35 cm.

7. Turn to page 307 of the textbook and answer exercise 1.e. of "Exercises: Checking Your Skills."

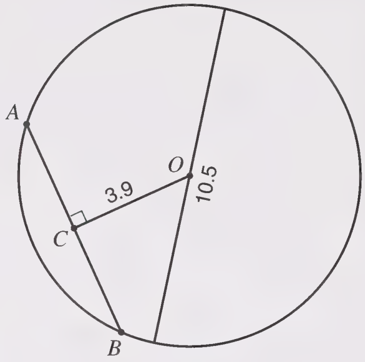
Compare your response with the suggested answer in the Appendix, Activity 2, page 61.



Turn to pages 305 and 306 of the textbook and work through "Example 2: Determine the length of a line segment in a circle." Then work through the example that follows.

Example

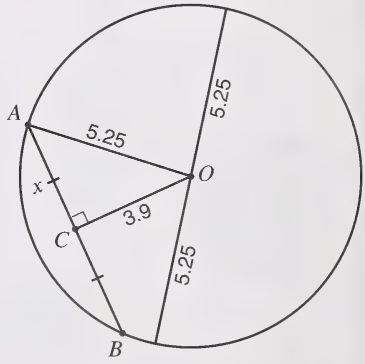
Determine the length of chord *AB* in the following diagram. Round your answer to 2 decimal places.



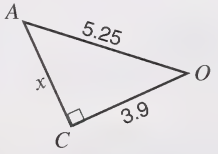
Solution

The diagram shows that the diameter of the circle is 10.5. Therefore, the radius is half of 10.5, or 5.25.

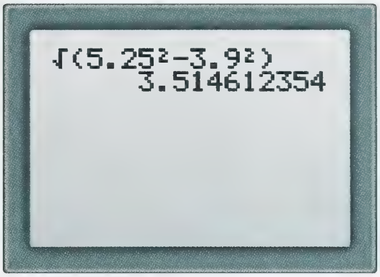
Draw radius *OA*. The diagram now looks like the one on the right.



Segments *AC* and *CB* are equal because a perpendicular from the centre of the circle to a chord bisects the chord. The following diagram shows how the Pythagorean Theorem can be applied.



$$\begin{aligned}
\therefore\ (OA)^2 &= (AC)^2 + (OC)^2 \\
5.25^2 &= x^2 + 3.9^2 \\
x^2 &= 5.25^2 - 3.9^2 \\
x^2 &= 27.5625 - 15.21 \\
x^2 &= 12.3525 \\
x &= \sqrt{12.3525} \\
&\doteq 3.51
\end{aligned}$$



```
√(5.25²-3.9²)
     3.514612354
```

The length of *AC* is about 3.51. Because *x* is half of the length of chord *AB*, the length of chord *AB* must be about 7.02.

8. Turn to page 307 of the textbook and answer exercise 1.d. of “Exercises: Checking Your Skills.”

9. Turn to pages 306 and 307 of the textbook and answer exercises 1 to 5 of “Discussing the Ideas.”

10. Turn to pages 307 to 310 of the textbook and answer the following.

- **a.** exercises 1.a., 1.b., 1.c., 1.f., 1.g., 1.h., 3, 4, 6, and 8 of “Exercises: Checking Your Skills”
- **b.** exercise 10 of “Exercises: Extending Your Thinking”

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 62–70.

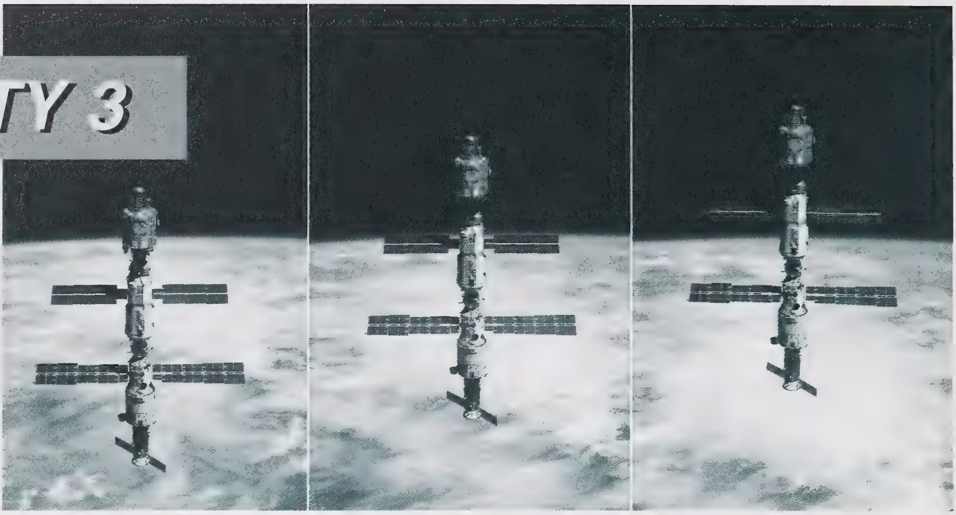
LOOKING BACK

In this activity you identified two properties of chords of a circle. You then applied these two properties to problems involving the lengths of segments within a circle.

11. Turn to page 311 of the textbook and answer “Communicating the Ideas.”

Compare your response with the suggested answer in the Appendix, Activity 2, page 71.

ACTIVITY 3



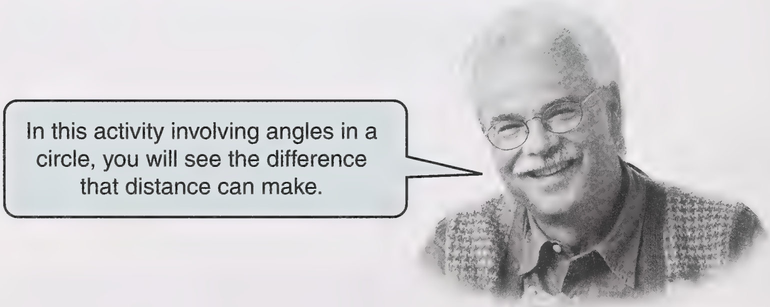
NASA

Properties of Angles in Circles

> *Each problem that I solved became a rule which served afterwards to solve other problems.*
>
> **—René Descartes (1596–1650)**

The sequence of pictures of the Mir space station were taken as the space shuttle circled the station to check its exterior. The changing point of view of Mir can be seen against the background of Earth. Notice how Mir takes up the same angle of view in each photograph as the space shuttle circles it.

You will see that there are many ways of getting the same part of a scene into a picture. You can change the lens or you can change the distance from the scene. If you have used a camera with a zoom lens, you will have experienced the changing angle of view caused by changing the focal length of a lens of a camera.

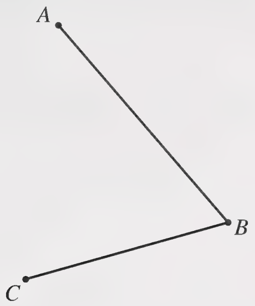

In this activity involving angles in a circle, you will see the difference that distance can make.

Getting Ready

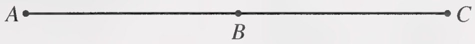
To prepare for your work with angles in circles, answer the following questions.

1. What is the measure of the following angles?

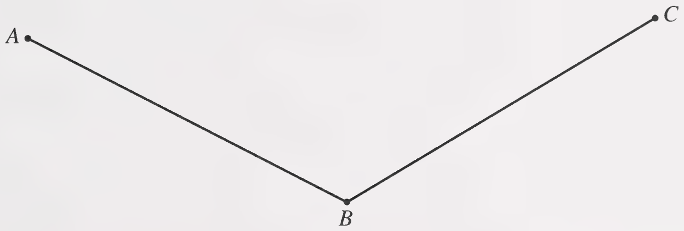
a.



b.



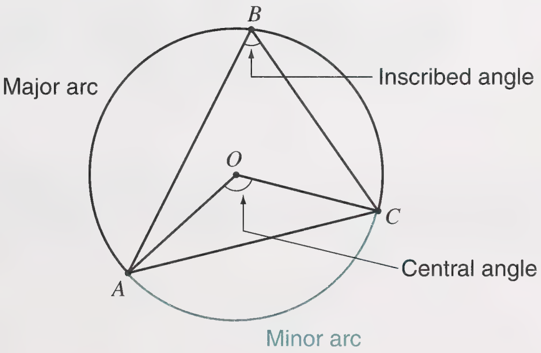
c.



2. What is a right angle? an acute angle? an obtuse angle?

Compare your responses with the suggested answers in the Appendix, Activity 3, page 71.

In this activity you are going to be working with angles in circles like those in the following diagram. The **minor arc** defined by chord AC is shown in colour, and the **major arc** defined by chord AC is shown in black. An **inscribed angle** defined by the minor arc is labelled $\angle ABC$, and the **central angle** defined by the minor arc is labelled $\angle AOC$.



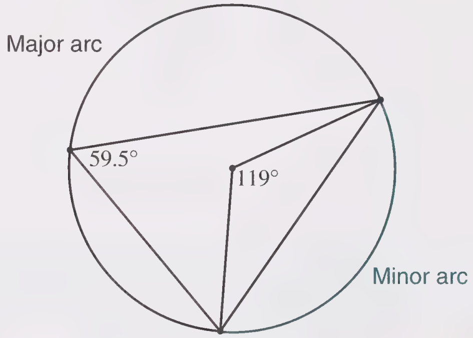
Turn to page 312 of the textbook and read the introductory paragraphs of Tutorial 6.3, "Properties of Angles in Circles."

If you are using interactive geometry software, do exercise 3. If you are not using interactive geometry software, do exercise 4.

3. Turn to pages 313 and 314 of the textbook and do the following.

- **a.** Complete Steps 1 to 4 of "Investigation 1: Central Angle and Inscribed Angle," answering exercises 11 and 12.
- **b.** Complete Step 5 of "Investigation 1: Central Angle and Inscribed Angle," answering exercise 14.

4. Draw three circles with a radius of 5 cm. Draw a chord that is not a diameter in each circle. Mark the major and minor arcs defined by the chords in different colours. For each minor arc, draw the central angle and an inscribed angle; then measure the six angles (two from each circle). A sample diagram is given.



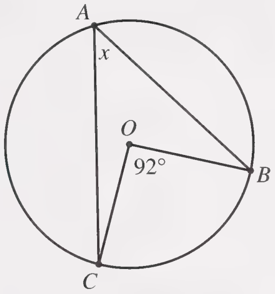
- **a.** Can you find a relationship between the central angle and the inscribed angle in each circle?
- **b.** Make a conjecture about the relationship of all such central and inscribed angles in all possible circles.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 71–72.

The conjecture regarding the central angle and an inscribed angle subtended by the same arc can be used to determine the measure of the central angle or the inscribed angle given one of the angles.

Example

Find the value of *x* (in degrees) in the following diagram.

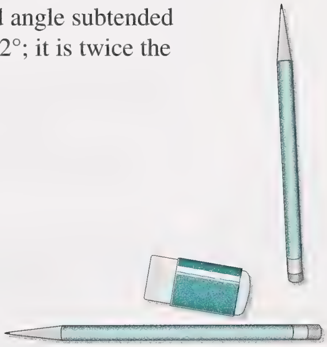


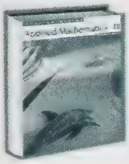
Solution

The central angle is twice the measure of an inscribed angle subtended by the same arc. In the diagram, the central angle is 92°; it is twice the measure of *x*. Thus, *x* will be half of 92°, or 46°.

You could also write this out algebraically.

$$\begin{aligned} 2(\angle CAB) &= \angle COB \\ 2x &= 92^\circ \\ \frac{2x}{2} &= \frac{92^\circ}{2} \\ x &= 46^\circ \end{aligned}$$





5. Turn to page 319 of the textbook and answer exercises 1.a., 1.d., and 1.f. of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 3, page 72.

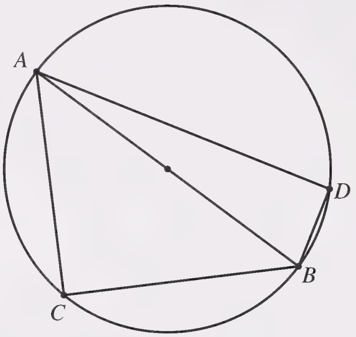
If you are using interactive geometry software, do exercise 6. If you are not using interactive geometry software, do exercise 7.

6. Turn to pages 315 and 316 of the textbook and do the following.

   a. Complete Steps 1 to 3 of "Investigation 2: Angle in a Semicircle," answering exercises 9 and 10.

   b. Complete Step 4 of "Investigation 2: Angle in a Semicircle," answering exercise 12.

7. Draw three circles with a radius of 5 cm. Draw a diameter in each circle. For each diameter, draw an inscribed angle in both of the arcs. A sample diagram is given.

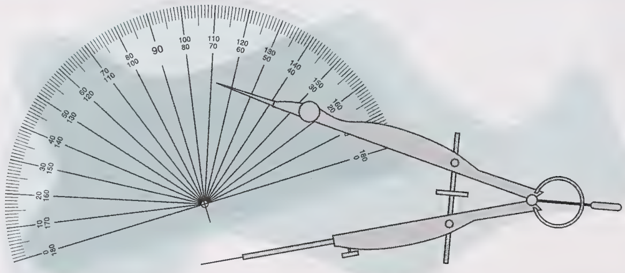


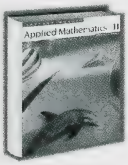
   a. What is the measure of each of the six angles (two from each circle) subtended by the diameters?

   b. Make a conjecture about the relationship of all such inscribed angles subtended by a diameter in all possible circles.

8. Turn to page 319 of the textbook and answer exercise 1.b. of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 3, page 72.





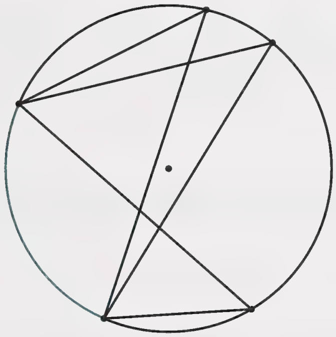
If you are using interactive geometry software, do exercise 9. If you are not using interactive geometry software, do exercise 10.

9. Turn to pages 316 and 317 of the textbook and do the following.

   a. Complete Steps 1 to 3 of "Investigation 3: Inscribed Angles," answering exercises 7 and 8.

   b. Complete Step 4 of "Investigation 3: Inscribed Angles," answering exercise 10.

10. Draw three circles with a radius of 5 cm. Mark two points on each circle. Avoid choosing the points that will make the segment joining them a diameter. For each minor arc, make three inscribed angles along the major arc; then measure these angles, and record your results. A sample diagram is given.



   a. Do you notice anything about the three angles in each circle?

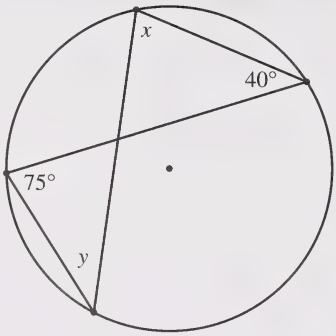
   b. Write a conjecture about the measures of inscribed angles subtended by the same arc in a circle.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 72–73.

The conjecture regarding the measure of inscribed angles subtended by the same arc can be used to determine the values of unknown angles.

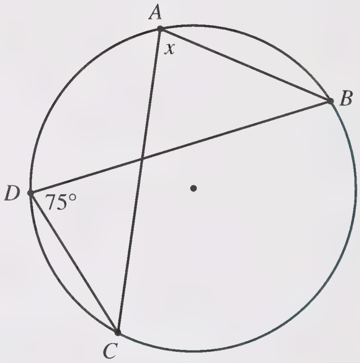
Example

Find the values of x and y (in degrees) in the following diagram.



Solution

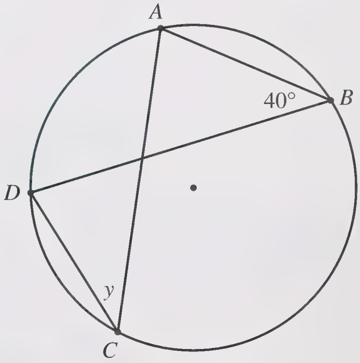
First, find the value of x. Make a modified diagram (like the one on the right), eliminating the extraneous information and highlighting the important information.



The diagram, drawn this way, shows that $\angle CAB$ and $\angle CDB$ are subtended by arc BC. Inscribed angles subtended by the same arc are of equal measure.

Therefore, $x = 75°$.

Now, find the value of y. Make a modified diagram (like the one on the right), again eliminating the unnecessary information and highlighting the important information.



The diagram, drawn this way, shows that $\angle DCA$ and $\angle DBA$ are subtended by arc AD. Inscribed angles subtended by the same arc are of equal measure.

Therefore, $y = 40°$.

When looking at complicated diagrams, you must focus on the items you need to solve the problem. Redrawing a diagram to include only the parts you need may make solving problems much easier.

Turn to page 318 of the textbook and work through "Example: Use properties of angles in circles to find an angle." You will see several useful techniques you can apply when solving questions.

11. Turn to pages 319 to 321 of the textbook and answer exercises 1.c., 1.e., 2, 3, 4, and 6 of "Exercises: Checking Your skills."

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 73–76.

LOOKING BACK

In this activity, you examined the following properites.

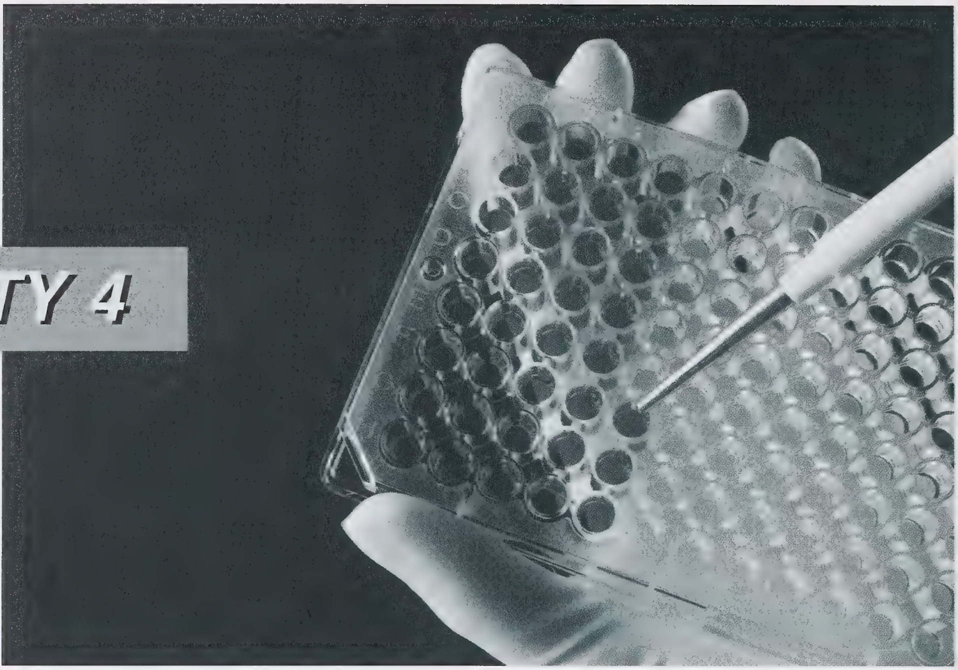
Properties of Angles in a Circle

- *The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.*
- *An angle inscribed in a semicircle is a right angle.*
- *Inscribed angles subtended by the same arc of a circle are equal.*

12. Turn to page 321 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 3, page 77.

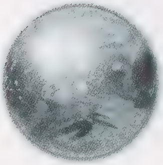
ACTIVITY 4

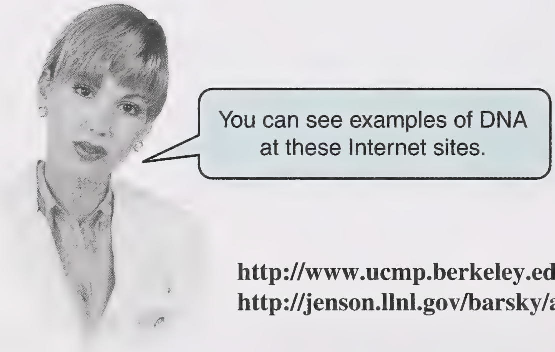


Properties of Tangents to a Circle

Inspiration is needed in geometry, just as much as in poetry.
—Aleksandr Sergeyevich Pushkin (1799–1837)

The world of biotechnology is full of wonderful patterns. Some of the patterns are natural, like the DNA of plants and animals.

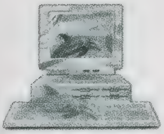


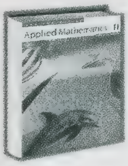

You can see examples of DNA at these Internet sites.

http://www.ucmp.berkeley.edu/glossary/gloss3/dna.html
http://jenson.llnl.gov/barsky/abasic_dynamic.html

Other patterns, like the one in the preceding picture, are used in items of human design: rows and columns of circles with small lines of materials separating them. In this activity you will examine several interesting relationships among circles and lines that touch them.

Turn to page 324 of the textbook and read the introductory paragraphs of Tutorial 6.4, "Properties of Tangents to a Circle."

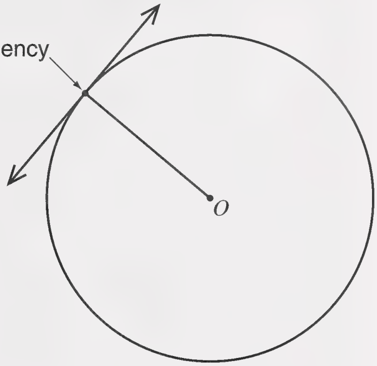




If you are using interactive geometry software, do exercise 1. If you are not using interactive geometry software, do exercise 2.

1. Turn to pages 324 to 326 of the textbook and complete Steps 1 to 5 of "Investigation 1: Angle Between a Tangent Line and Radius," answering exercises 10, 11, and 13. **Note:** You may need to try this construction several times.

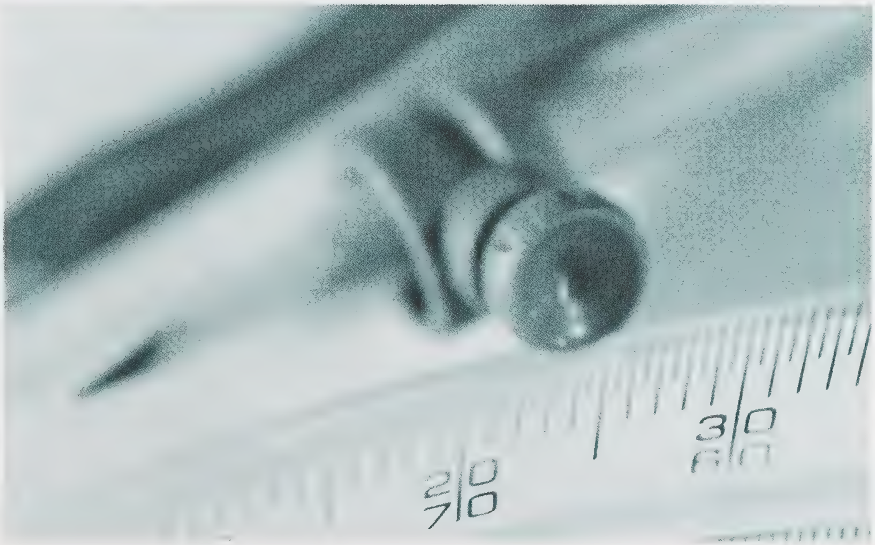
2. Construct three circles with radii of 3 cm, 4 cm, and 5 cm. On each circle, pick a point on the circle and draw a radius to the selected point. Draw a line that passes through the selected point on the circle but through no other points on the circle. (This is known as a tangent line.)



   a. Measure the angles between the tangent line and the radius in each circle. What do you notice?

   b. Write a conjecture about the angle between a tangent line and a radius at the point of tangency.

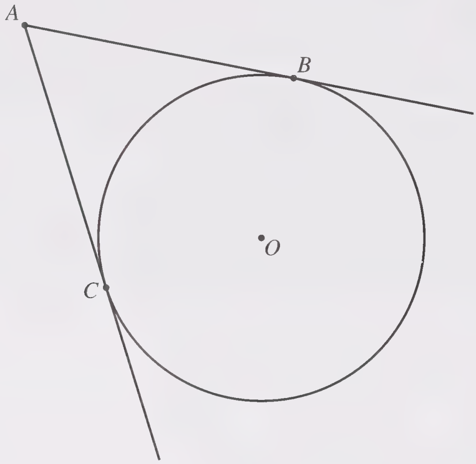
Compare your responses with the suggested answers in the Appendix, Activity 4, page 77.



If you are using interactive geometry software, do exercise 3. If you are not using interactive geometry software, do exercise 4.

3. Turn to pages 326 to 328 of the textbook and complete Steps 1 to 4 of "Investigation 2: The Lengths of Tangent Segments from an External Point," answering exercises 8, 9, 10, and 12.

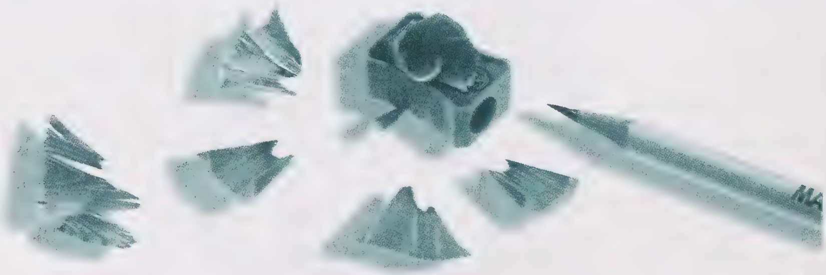
4. Construct three circles with radii of 3 cm, 4 cm, and 5 cm. For each circle, pick a point outside the circle and draw two tangents to the circle from the chosen point.



a. For each of the tangents, measure the distances from point *A*, outside the circle, to the points of tangency. What do you notice?

b. Write a conjecture about the lengths of the tangent segments to a circle from an external point.

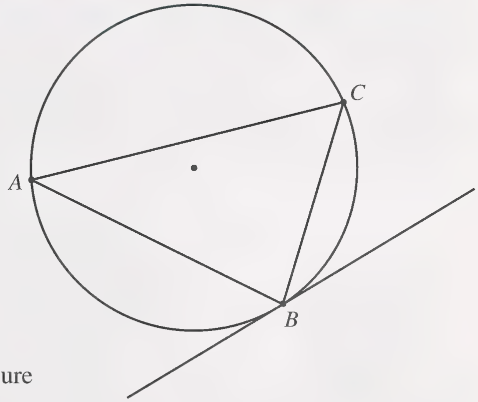
Compare your responses with the suggested answers in the Appendix, Activity 4, pages 77–78.



If you are using interactive geometry software, do exercise 5. If you are not using interactive geometry software, do exercise 6.

5. Turn to pages 328 to 330 of the textbook and complete Steps 1 to 6 of "Investigation 3: The Angle Between a Tangent and a Chord and the Inscribed Angle on the Opposite Side of the Chord," answering exercises 11, 12, and 14.

6. Construct three circles with radii of 3 cm, 4 cm, and 5 cm. For each circle, draw a chord that splits the circle into a major and minor arc; draw a tangent to the circle through one of the chord's endpoints; and draw an inscribed angle subtended by the chord.



   a. For each diagram, measure the inscribed angle and the angle between the tangent and the chord. What do you notice about the measure of these angles?

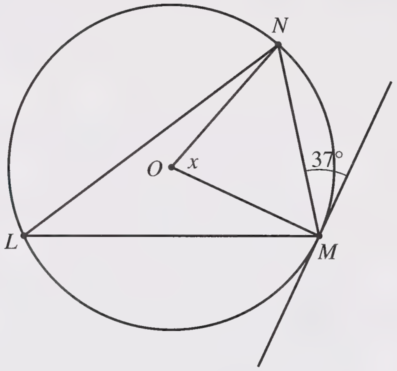
   b. Write a conjecture about the measure of the angle between a tangent and a chord and the inscribed angle that is opposite the chord.

Compare your responses with the suggested answers in the Appendix, Activity 4, page 78.

The tangent properties you have discovered in this activity can be used to determine the measures of angles in a circle.

Example

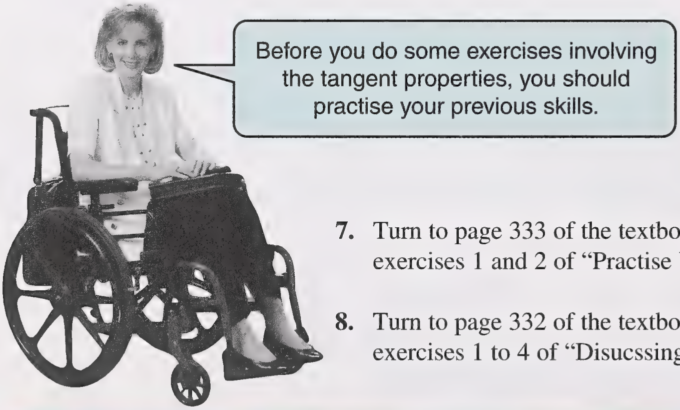
Determine the value of x in the following diagram. The centre of the circle is O.



Solution

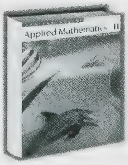
$\angle MLN = 37°$ because it is the inscribed angle on the opposite side of the chord from the tangent.

$\angle MLN$ is subtended by the minor arc MN, and x is a central angle subtended by the same arc. Therefore, x is twice $\angle MLN$, or 74°.

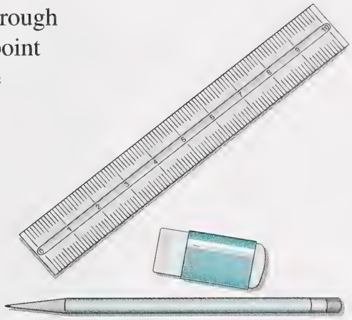

Before you do some exercises involving the tangent properties, you should practise your previous skills.

7. Turn to page 333 of the textbook and answer exercises 1 and 2 of "Practise Your Previous Skill."
8. Turn to page 332 of the textbook and answer exercises 1 to 4 of "Disucssing the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 78–80.



Turn to pages 331 and 332 of the textbook and work through "Example 1: Determine the distance from an external point to the centre of a circle" and "Example 2: Calculate the length of a tangent segment."



9. Answer exercises 1.a., 1.b., 1.c., 1.f., 2, 3.a., 3.c., 3.e., 3.f., and 6 of "Exercises: Checking Your Skills" on pages 333 to 336 of the textbook. Round your answers to 2 decimal places where necessary.

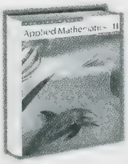
Compare your responses with the suggested answers in the Appendix, Activity 4, pages 81–85.

LOOKING BACK

In this activity, you examined the following properites.

Properties of Tangents to a Circle

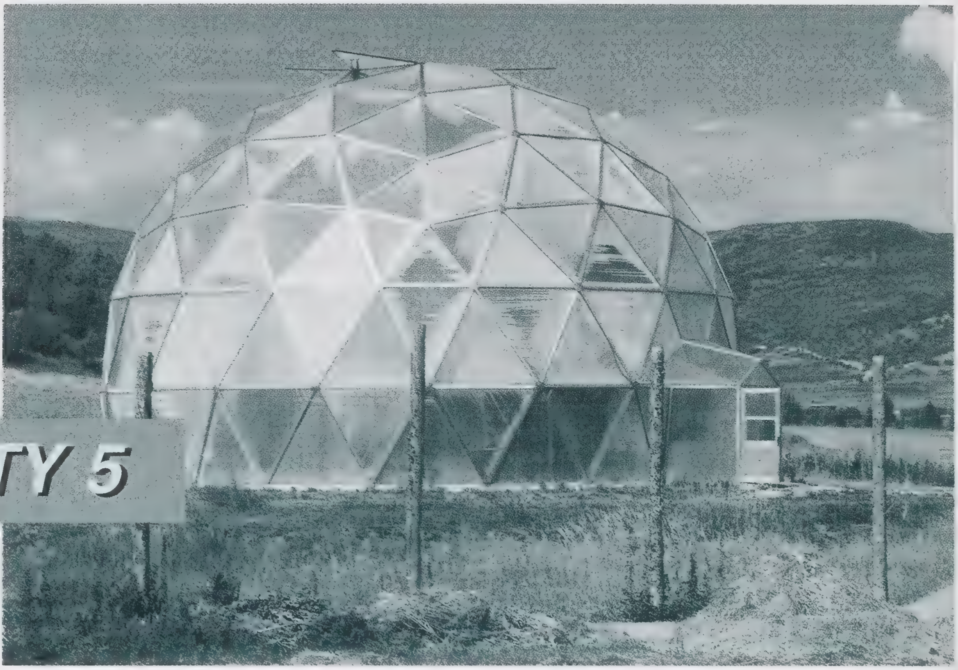
- *A tangent to a circle is perpendicular to the radius at the point of tangency.*
- *The tangent segments to a circle from an external point are equal.*
- *The angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord.*



10. Turn to page 337 of the textbook and answer exercise 9 of "Exercises: Extending Your Thinking." Round the final answer to 2 decimal places.

11. Turn to page 337 of the textbook and answer "Communicating the Ideas."

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 86–87.

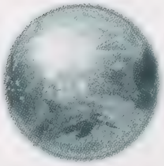


ACTIVITY 5

Properties of Polygons

Where there is matter, there is geometry.
—Johannes Kepler (1571–1630)

A geodesic dome is an intriguing three-dimensional object. It seems so fragile, yet is among the stongest structures that humans can build. The photo is of a fairly small geodesic dome, but it still shows the main characteristics very clearly. The shape approximates a part of a sphere. In fact, if a geodesic dome is built accurately, it can be circumscribed by a sphere. A geodesic dome is built of triangles that make up various pyramidal shapes that cover the entire structure.



For information on how geodesic domes can be constructed, you may wish to check the following Internet site:

http://www.desertdomes.com/tips.html

Some of the famous geodesic domes have Internet sites showing their pictures. You might want to check out these sites:

http://naid.sppsr.ucla.edu/expo67/map-docs/unitedstates.htm
http://montreal.cityvu.com/visions/vmtl6203.jpg

In this activity you will investigate some properties of polygons and develop a formula to determine the sum of the interior angles of any polygon. You will identify some properties of cyclic and non-cyclic quadrilaterals and analyse how triangles can be used to build an interesting polygon known as the Koch snowflake.

Polygons can have many sides and angles. To determine the sum of the interior angles of a polygon, you need to develop a formula. You can develop this formula by studying the pattern of the sum of the interior angles in polygons with increasing numbers of sides.

1. Turn to pages 338 and 339 of the textbook and answer exercises 1 to 3 of "Investigation 1: Interior Angles in a Polygon."

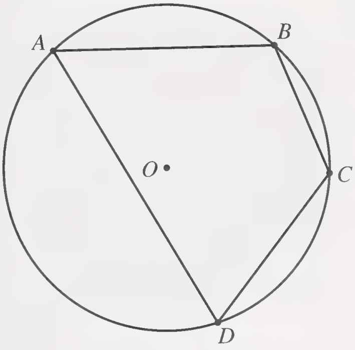
Compare your responses with the suggested answers in the Appendix, Activity 5, pages 88–90.

One particular polygon of interest is the cyclic quadrilateral. Turn to page 339 of the textbook and study the definition of a cyclic quadrilateral.

If you are using interactive geometry software, do exercise 2. If you are not using interactive geometry software, do exercise 3.

2. Turn to page 340 of the textbook and complete Steps 1 to 4 of "Investigation 2: Cyclic Quadrilaterals," answering exercises 5, 6, 7, and 9.

3. Construct three circles with radii larger than 6 cm each. Mark four points on the first circle. Join these points to form a cyclic quadrilateral.



   a. Measure each angle of the quadrilateral; then find the sum of each pair of opposite angles.

   b. Predict what the sum of the opposite angles in other cyclic quadrilaterals will be. Repeat the process with your other two circles making differently shaped quadrilaterals. Is your prediction correct?

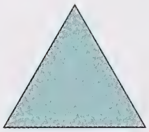
   c. Write a conjecture about the opposite angles in a cyclic quadrilateral.

Compare your responses with the suggested answers in the Appendix, Activity 5, page 91.

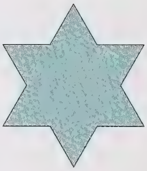
Turn to page 341 of the textbook and study the summary of the investigations at the top of the page. Then work through "Example: Sum of the Interior Angles of a Polygon" on pages 341 and 342 of the textbook.

The Koch snowflake is a most interesting polygon. While the area of the polygon doesn't change much, the perimeter is boundless. If you build enough triangles on triangles on triangles, the perimeter can be made as large as you want.

For example, if you start with an equilateral triangle with sides 10 cm long, the initial perimeter is 30 cm. When the next step is taken, the perimeter is determined as follows:



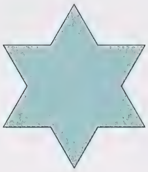
For each side of the polygon, divide it into three equal parts, remove the middle part, and add two new segments of the same length as the one removed. There are now 12 smaller segments, each $\frac{10}{3}$ cm long.



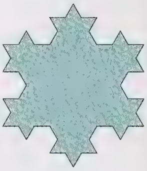
$$\therefore \overset{4}{\cancel{12}} \times \frac{10}{\underset{1}{\cancel{3}}} = 40$$

The perimeter is 40 cm.

When the next step is taken, the perimeter is determined as follows:



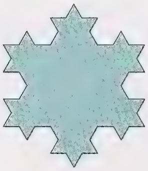
For each side of the polygon, divide it into three equal parts, remove the middle part, and add two new segments of the same length as the one removed. There are now 48 smaller segments, each $\frac{10}{9}$ cm long.



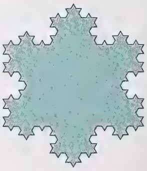
$$\begin{aligned}\therefore \overset{16}{\cancel{48}} \times \frac{10}{\underset{3}{\cancel{9}}} &= \frac{160}{3} \\ &\doteq 53\end{aligned}$$

The perimeter is about 53 cm.

When the next step is taken, the perimeter is determined as follows:



For each side of the polygon, divide it into three equal parts, remove the middle part, and add two new segments of the same length as the one removed. There are now 192 smaller segments, each $\frac{10}{27}$ cm long.



$$\begin{aligned}\therefore \overset{64}{\cancel{192}} \times \frac{10}{\underset{9}{\cancel{27}}} &= \frac{640}{9} \\ &\doteq 71\end{aligned}$$

The perimeter is about 71 cm.

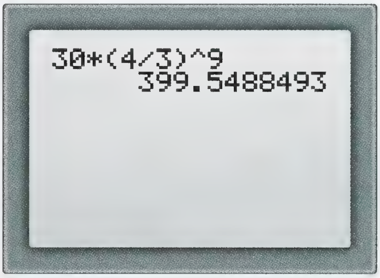
Actually, the perimeter increases by a factor of $\frac{4}{3}$ at each step. A summary of the first few steps are shown in the following chart.

Step	1	2	3	4	5	6
Calculation	30	$\frac{4}{3} \times 30$	$\left(\frac{4}{3}\right)^2 \times 30$	$\left(\frac{4}{3}\right)^3 \times 30$	$\left(\frac{4}{3}\right)^4 \times 30$	$\left(\frac{4}{3}\right)^5 \times 30$
Perimeter (cm)	30	40	$53.\overline{3}$	$71.\overline{1}$	$94.\overline{814}$	126.420

What is the perimeter after 10 steps? after 20 steps? after 100 steps?

After 10 Steps

To calculate the perimeter after 10 steps, you need to multiply the original perimeter by $\frac{4}{3}$ nine times, or by $\left(\frac{4}{3}\right)^9$.

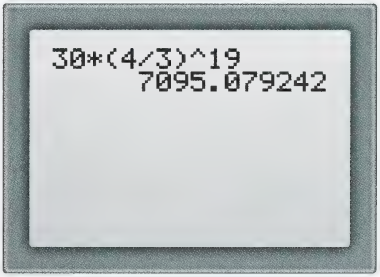


```
30*(4/3)^9
         399.5488493
```

The perimeter after 10 steps is approximately 399.5 cm.

After 20 Steps

To calculate the perimeter after 20 steps, you need to multiply by $\frac{4}{3}$ nineteen times, or by $\left(\frac{4}{3}\right)^{19}$.

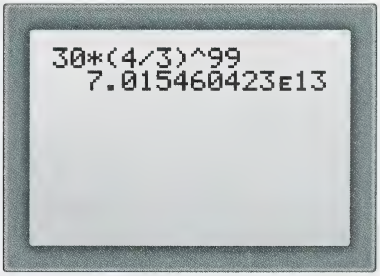


```
30*(4/3)^19
         7095.079242
```

The perimeter after 20 steps is approximately 7095.1 cm.

After 100 Steps

To calculate the perimeter after 100 steps, you need to multiply by $\frac{4}{3}$ ninety-nine times, or by $\left(\frac{4}{3}\right)^{99}$.

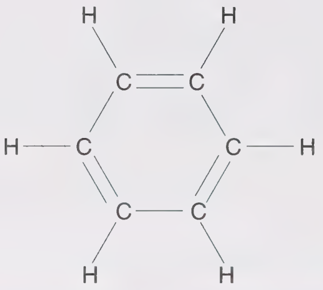


```
30*(4/3)^99
      7.01546042 3E13
```

The perimeter after 100 steps is approximately 7.0×10^{13} cm.

4. Turn to page 342 of the textbook and answer exercises 1 to 4 of "Discussing the Ideas."

5. Turn to pages 343 and 344 in your textbook and answer exercises 1 to 5 of "Exercises: Checking Your Skills." For exercise 5, you might find the following diagram of a benzene ring useful.



Compare your responses with the suggested answers in the Appendix, Activity 5, pages 91–95.

LOOKING BACK

In this activity, you examined two polygon properties.

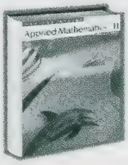
Polygon Properties

- *The opposite angles of a cyclic quadrilateral are supplementary; that is, the sum of the two angles is 180°.*
- *The sum of the angles of any polygon with n sides is $180°(n-2)$, or $(2n-4)$ right angles.*

6. Turn to page 345 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 95.

Follow-up Activities



This module dealt with Chapter 6: Circles of the *Addison-Wesley Applied Mathematics 11 Source Book*. Turn to page 348 of the textbook and review the skills and concepts that were developed in this module. Also, read the important results and formulas you discovered.

Answer exercises 2.a., 2.c., 3, 4, 5.c., 6.a., 6.d., 7.b., 7.d., 8, 11, and 12 of Part B of "What Should I Be Able to Do?" on pages 349 to 352 of the textbook.

Compare your responses with the suggested answers in the Appendix, Follow-up Activities, pages 96–101.

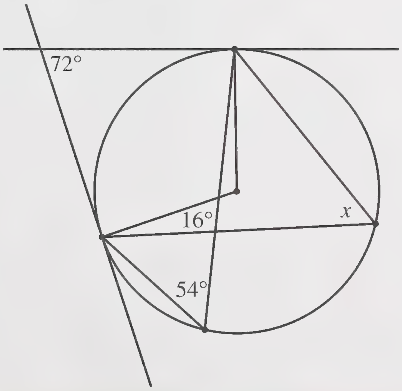
If you had difficulties understanding the skills and concepts in Module 6: Circles, it is recommended that you do the Extra Help. If you have a clear understanding of the skills and concepts in this module, it is recommended that you do the Enrichment. You may decide to do both.

EXTRA HELP

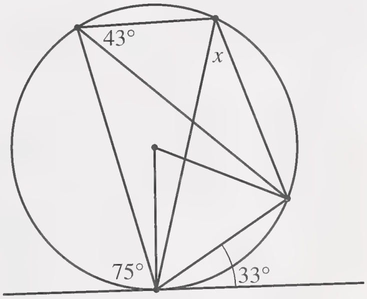
In this module, you explored properties of circles and angles and lines in circles. You should be able to recognize the properties that you studied when you see them in a diagram or a real-world situation. Often, the key to recognizing what property can be found is to remove information that gets in the way of noticing the key to the situation.

Look at the following diagrams. Redraw the diagrams and remove the information that is not needed to find the value for x.

1.



2.



Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Extra Help, page 102.

ENRICHMENT

There are many more properties involving chords and circles you have not covered yet. The following Internet site introduces Euclid's *Elements*—13 books consisting of the logical development of geometry and other branches of mathematics:

http://aleph0.clarku.edu/~djoyce/java/elements/elements.html

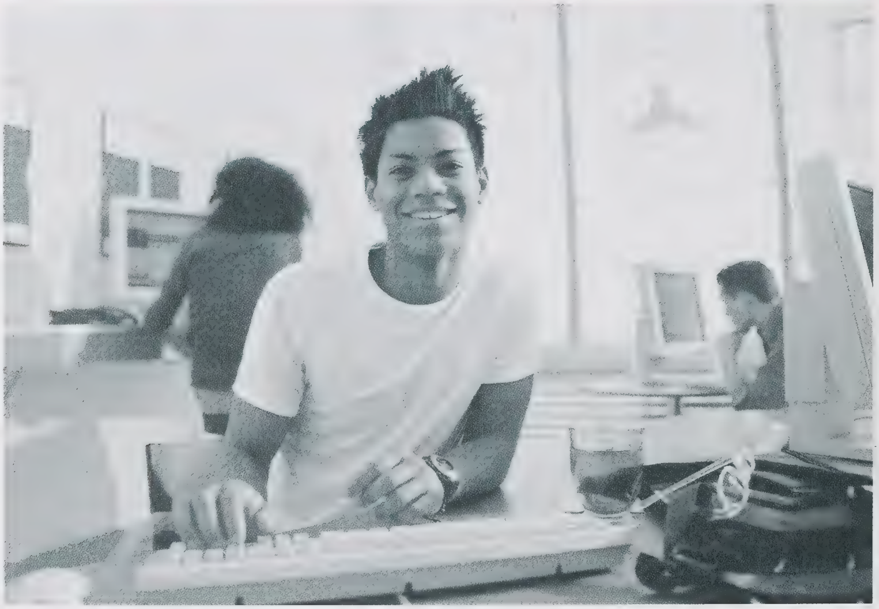
Book III of Euclid's *Elements* contains information regarding circles and chords.

1. Turn to page 349 of the textbook and work through Part A of "What Should I Be Able to Do?," answering exercises 1.d., 1.e., and 1.g. Be sure to work with at least three different circles before you make your conjecture.

> If you are not using interactive geometry software, draw the circles and chords using a compass and ruler and measure the lines with your ruler.

2. Turn to page 345 of the textbook and do exercise 7 of "Exercises: Extending Your Thinking."

Compare your responses with the suggested answers in the Appendix, Follow-up Activities: Enrichment, page 102.



MODULE PROJECT

CIRCLE DESIGN

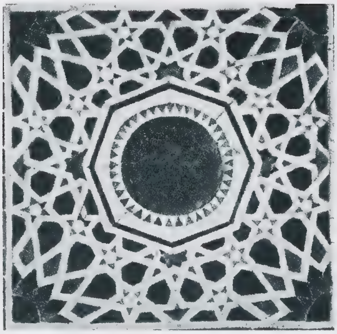
Completing the Project

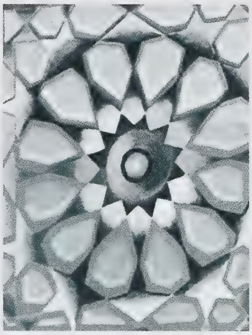
Imagination is more important than knowledge.
—Albert Einstein (1879–1955)

By now you should have completed the initial research into your circle design. You are to create a circle design of your own. You may complete a design similar to the Aboriginal designs or you may want to create an original circle design of your own. Using a computer drawing program is acceptable as is making the design by hand.

Turn to page 346 of the textbook and read "The Details." You will find out exactly what you need to do to complete your project.

The following illustrations are examples of other designs possessing some circles and their properties. You may be able to use many more of the properties studied in this module in your design than those shown in these designs.





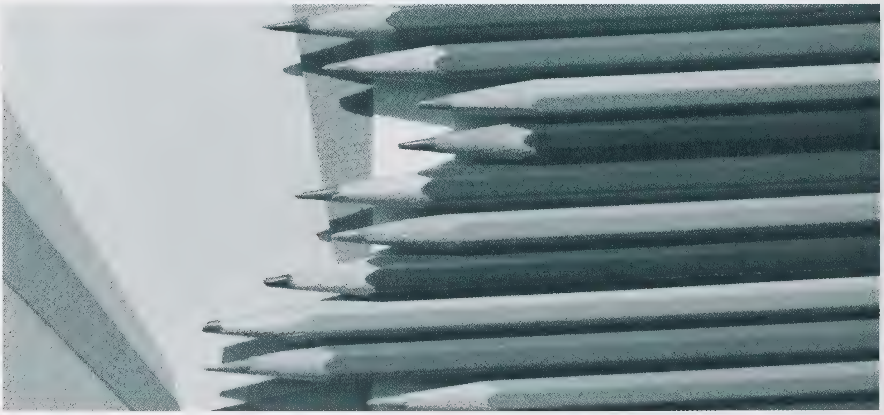
Turn to pages 352 to 354 of the textbook and answer exercises 1 and 2 of Part C of "What Should I Be Able to Do?" These exercises will help you to understand what you should be including when you submit your project for marking.

Compare your responses with the suggested answers in the Appendix, Module Project, page 102.

Module Project

Now that you have seen the kind of work shown in the sample project work on pages 352 to 354 of the textbook, you may want to revise or enhance some of your thoughts about your design. Then take out the Project Booklet that accompanies this Student Module Booklet and complete the module project, Circle Design.

Submit your completed Module 6 Project Booklet to your teacher.

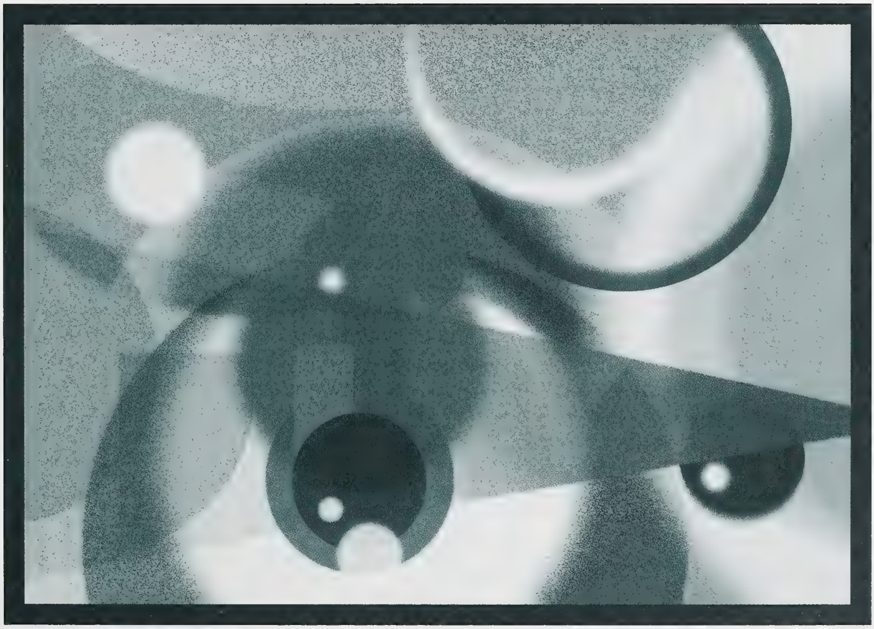


Module Summary

In this module you discovered a number of properties of circles, chords, angles, tangents, and polygons. You examined perpendicular bisectors and chords of a circle, central angles and inscribed angles subtended by the same arc, tangents to a circle from an external point, the sum of angles in a polygon, and angles in a cyclic quadrilateral. You used interactive geometric software or a compass, ruler, and protractor to explore these interesting and useful facts about circle geometry.

Knowing and understanding some of the properties of circles is very useful when you study how many objects, pictures, and designs use circles as their basis.

If you are interested in design or art, you will find your knowledge of the properties of circles a definite asset when completing projects involving various designs. You will be able to draw interesting and attractive patterns as well as interpret unique art forms.



Module Assignment

To demonstrate what you have learned in this module, complete the module assignment in the Assignment Booklet.

Submit your completed Module 6 Assignment Booklet to your teacher.

APPENDIX

Glossary
Suggested Answers
Image Credits

GLOSSARY

acute angle: an angle measuring less than 90°

arc: part of a circle

axiom: a statement assumed to be true

bisect: to divide into two equal parts

central angle: an angle composed of two radii of a circle

centre: middle point

chord: a line segment whose endpoints are on the circumference of a circle

circle: a figure bounded by all points equidistant from a fixed point called the centre

circumcircle: a circle that includes the vertices of a plane figure

circumference: the perimeter of a circle

complementary angles: a pair of angles whose sum is 90°

conjecture: a supposition or guess; a generalization of a set of observations

diameter: a chord that passes through the centre of the circle

hypotenuse: the longest side of a right triangle

inscribed angle: an angle formed by two chords of a circle

intersect: to meet or cross

line segment: the portion of a line between two points

major arc: the larger of the two arcs created by a chord of a circle

measure: the size of an object expressed using a standard unit

minor arc: the smaller of the two arcs created by a chord of a circle

obtuse angle: an angle measuring more than 90° but less than 180°

perpendicular: form a right angle

pi: the ratio of the circumference of a circle to the circle's diameter

point: a concept of position with no magnitude

polygon: a shape bounded by line segments

quadrilateral: a four-sided polygon

radii: plural form of radius

radius: any line joining a point on a circle to the centre of the circle

rectangle: a four-sided polygon with each angle a right angle

regular polygon: a polygon with equal sides and angles

right angle: an angle measuring 90°

semicircle: half of a circle

subtend: to give rise to

supplementary angles: a pair of angles whose sum is 180°

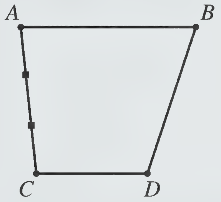
tangent line: a line that touches the curve or circle at only one point

theorem: a statement that can be proven from a set of axioms

Activity 1: Using Interactive Geometry Software

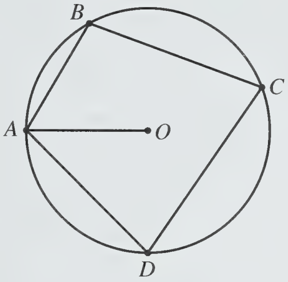
1. Textbook exercises 1 and 2 of "Investigation 1: The Toolbox," pp. 294 and 295

1. You should have seen the buttons as displayed on page 294 of the textbook.
2. Your quadrilateral will look something like the following.



Your circle should be similar to the one on page 295 of the textbook.

2. Diagrams will vary. A sample diagram is given.



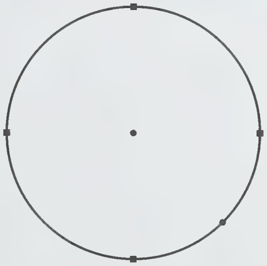
3. Textbook exercises 1 to 3 of "Investigation 2: The Measure Menu," pp. 296 to 298

1. Measurements will vary. Your display should look somewhat like the following.

Radius $\odot AB = 1.94$ cm

Area $\odot AB = 11.82\text{ cm}^2$

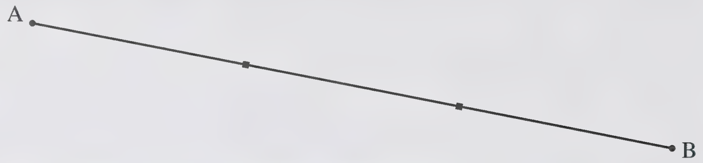
Circumference $\odot AB = 12.19$ cm



Activity 1 (continued)

2. Segments will vary. A sample segment and its measurement is given.

$m\,\overline{AB} = 10.00$ cm



3. Measurements will vary. A sample pentagon and its measurements are given.

$m\,\overline{AE} = 4.96$ cm

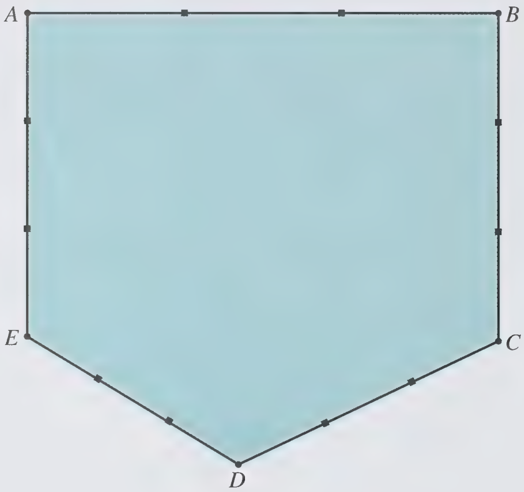
$m\,\overline{ED} = 3.79$ cm

$m\,\overline{DC} = 4.41$ cm

$m\,\overline{CB} = 5.03$ cm

$m\,\overline{BA} = 7.22$ cm

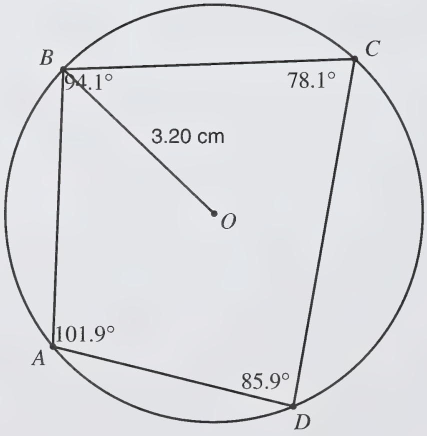
Area $p1 = 42.88\text{ cm}^2$



4. Diagrams will vary. A sample answer is given.

$$\begin{aligned} A &= \pi r^2 \\ &\doteq \pi(3.20)^2 \\ &\doteq \pi(10.24) \\ &\doteq 32.17 \end{aligned}$$

The area is approximately 32.17 cm^2.



5. Textbook exercises 1 to 4 of "Discussing the Ideas," p. 299

1. To select all the points on the screen at once, activate the Point tool and choose "Select All" from the Edit menu.
2. To deselect an object, either click on another object or in an empty space on the screen using the Selection arrow.
3. To change the shape of a triangle, click and drag one of the vertices of the triangle using the Selection arrow.
4. To change the size of a circle, click and drag the control point or the centre point of the circle using the Selection arrow.

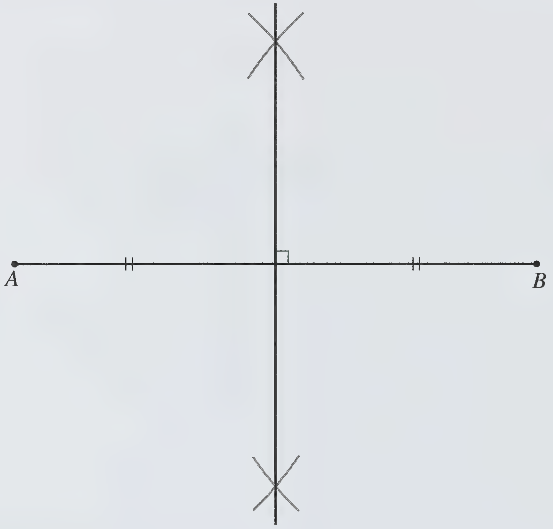
6. Textbook exercise "Communicating the Ideas," p. 299

Advantages	Disadvantages
• creates diagrams quickly and accurately	• have to learn new software package
• can do multiple examples quickly and easily	• less practise doing the geometric constructions by hand
• can quickly see what a change does to the whole diagram	• greater chance of forgetting how to do hand constructions
• can see patterns developing from the quickly changing diagrams	

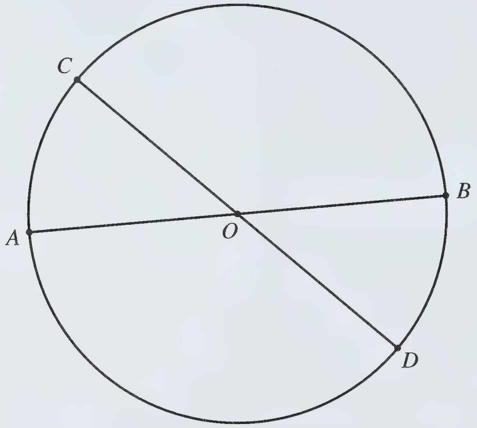
Activity 2: Properties of Chords in Circles

1. Textbook exercises 1 and 2 of "Practise Your Prior Skills," p. 300

1. Your diagram should be similar to the following.



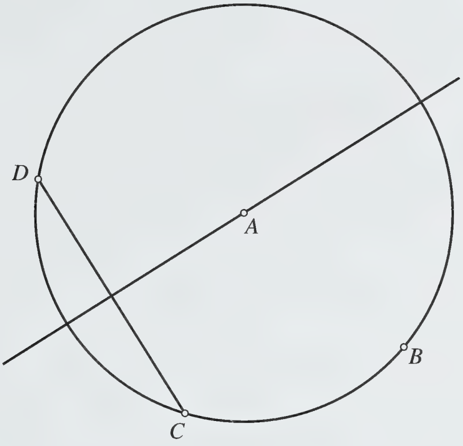
2. Your diagram should be similar to the following.



The chords are called diameters of the circle.

2. a. **Textbook exercise 6 of "Investigation 1: Chord Property 1," p. 301**

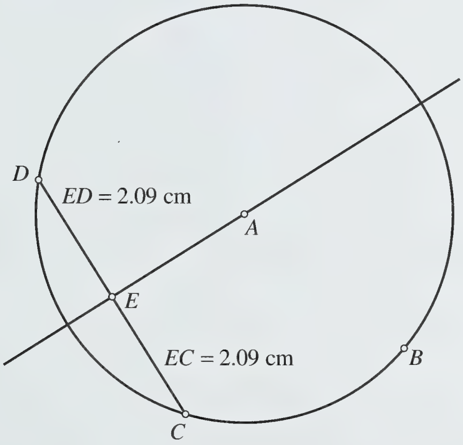
6. Diagrams will vary. A sample diagram is given.



The perpendicular line appears to pass through the midpoint of the chord.

b. **Textbook exercises 13 and 14 of "Investigation 1: Chord Property 1," p. 302**

13. The lengths of *EC* and *ED* are the same.



14. The distances from the point of intersection of the perpendicular to the endpoints of the chord should be the same for each chord. The point of intersection of the perpendicular and the chord is called the midpoint of the chord.

Activity 2 (continued)

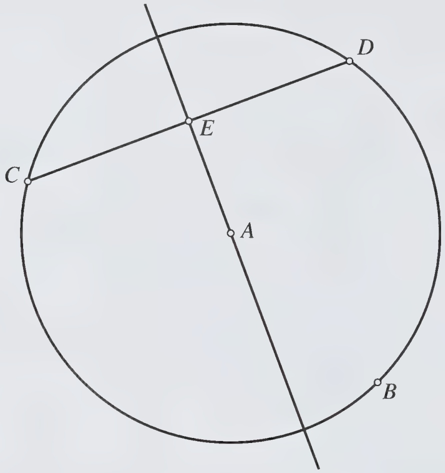
c. Textbook exercise 17 of "Investigation 1: Chord Property 1," p. 302

17. A line that passes through the centre of a circle and is perpendicular to a chord bisects that chord.

3. a. The lengths of the two segments of the chord are equal.
b. The lengths of the two segments of the chord would be equal.
c. A line that passes through the centre of a circle and is perpendicular to a chord bisects that chord.

4. a. Textbook exercises 7 and 8 of "Investigation 2: Chord Property 2," p. 303

7. Diagrams will vary. A sample diagram is given.



The perpendicular bisector of this chord passes through the centre of the circle.

8. The perpendicular bisector of a chord will pass through the centre of the circle.

b. Textbook exercise 11 of "Investigation 2: Chord Property 2," p. 303

11. The perpendicular bisector of a chord always passes through the centre of the circle.

5. a. The perpendicular bisector of a chord passes through the centre of the circle.

b. The perpendicular bisectors of two chords in any circle will meet at the centre of the circle.

6. Textbook exercises 1.a. and 1.c. of "Practise Your Prior Skills," p. 303

1. a.

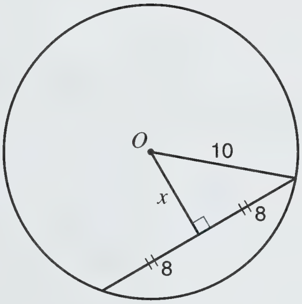
$$c^2 = a^2 + b^2$$
$$x^2 = 8^2 + 10^2$$
$$x^2 = 64 + 100$$
$$x^2 = 164$$
$$x = \sqrt{164}$$
$$\doteq 12.81$$

c.

$$c^2 = a^2 + b^2$$
$$13^2 = 7^2 + x^2$$
$$x^2 = 13^2 - 7^2$$
$$x^2 = 169 - 49$$
$$x^2 = 120$$
$$x = \sqrt{120}$$
$$\doteq 10.95$$

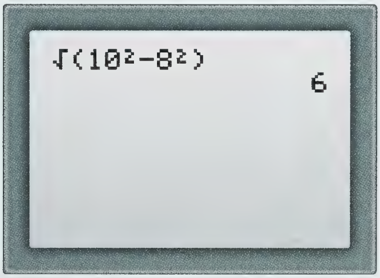
7. Textbook exercise 1.e. of "Exercises: Checking Your Skills," p. 307

1. e. The diagram shows that the radius of the circle is 10. Because the chord is 16, each of the two equal halves created by a perpendicular bisector is 8. The following diagram shows this new information.



From the diagram, notice that the Pythagorean Theorem can be used to solve for x.

$$c^2 = a^2 + b^2$$
$$10^2 = x^2 + 8^2$$
$$100 = x^2 + 64$$
$$x^2 = 100 - 64$$
$$x^2 = 36$$
$$x = \sqrt{36}$$
$$x = 6$$

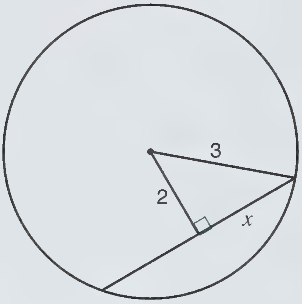


```
√(10²-8²)
                  6
```

Activity 2 (continued)

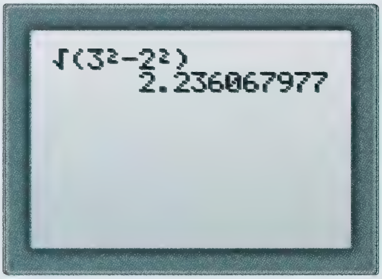
8. Textbook exercise 1.d. of "Exercises: Checking Your Skills," p. 307

1. **d.** The diagram shows that the radius of the circle is 3. Draw the diagram so the radius is drawn to the endpoint of the chord.



From the diagram, you can see that the Pythagorean Theorem can be used to solve for x.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 3^2 &= 2^2 + x^2 \\ 9 &= 4 + x^2 \\ x^2 &= 9 - 4 \\ x^2 &= 5 \\ x &= \sqrt{5} \\ &\doteq 2.24 \end{aligned}$$



```
√(3²-2²)
          2.236067977
```

9. Textbook exercises 1 to 5 of "Discussing the Ideas," pp. 306 and 307

1. Using a diameter in Investigation 1 would not have allowed you to construct a perpendicular from the centre to the chord, since the centre is part of a diameter. Using a diameter in Investigation 2 would have made arriving at a conjecture extremely difficult.

2. Because a perpendicular from the centre of a circle to a chord of the circle bisects the chord, C is the midpoint of chord AB.

3. Because the perpendicular bisector of a chord passes through the centre of the circle, the line through point M will also pass through point O, the centre of the circle.

4. a. Any line segment can have infinitely many circles passing through its endpoints. For one circle, the segment is a diameter. For all other circles, the segment is a chord.

   b. Because you do not know that the line bisects the segment and do not know that the line is perpendicular to the segment, you can draw no conclusions about the line.

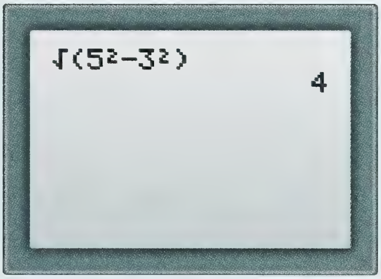
   c. A line segment whose endpoints are on a circle is called a chord.

5. The triangle formed is a right triangle. If you know the lengths of two sides of this triangle, the Pythagorean Theorem can be used to find the length of the third side.

10. a. Textbook exercises 1.a., 1.b., 1.c., 1.f., 1.g., 1.h., 3, 4, 6, and 8 of "Exercises: Checking Your Skills," pp. 307 to 310

1. a. The given diagram shows a right triangle. Use the Pythagorean Theorem to solve for x.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 5^2 &= 3^2 + x^2 \\ 25 &= 9 + x^2 \\ x^2 &= 25 - 9 \\ x^2 &= 16 \\ x &= \sqrt{16} \\ &= 4 \end{aligned}$$

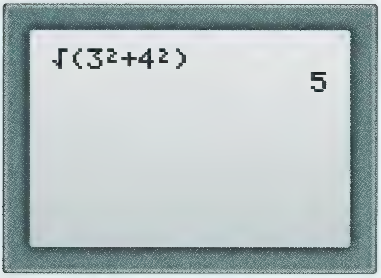


```
√(5²-3²)
                4
```

   b. The given diagram shows a right triangle with an unknown hypotenuse and one side of length 4. The third side is exactly one half the chord length, 6, since the segment from the centre of the circle to the chord is the perpendicular bisector of the chord.

   Use the Pythagorean Theorem to solve for x.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= 3^2 + 4^2 \\ x^2 &= 9 + 16 \\ x^2 &= 25 \\ x &= \sqrt{25} \\ &= 5 \end{aligned}$$



```
√(3²+4²)
                5
```

Activity 2 (continued)

c. The given diagram shows a right triangle with a hypotenuse of length 5 and a second side of length 3. This second side bisects the chord of the circle; therefore, the third side of the triangle has a length of $\frac{x}{2}$.

Use the Pythagorean Theorem to solve for x.

$$c^2 = a^2 + b^2$$

$$5^2 = 3^2 + \left(\frac{x}{2}\right)^2$$

$$25 = 9 + \frac{x^2}{4}$$

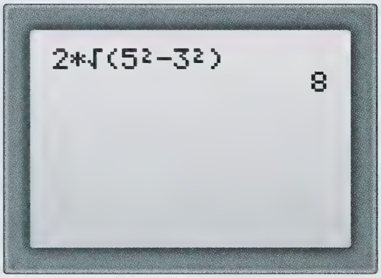
$$\frac{x^2}{4} = 25 - 9$$

$$\frac{x^2}{4} = 16$$

$$x^2 = 64$$

$$x = \sqrt{64}$$

$$= 8$$



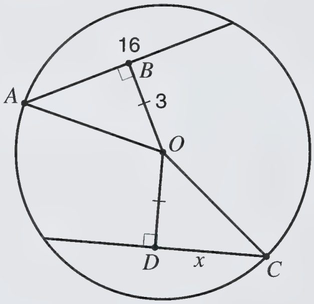
```
2*√(5²-3²)
                8
```

f. If you add two radii to the circle in the diagram and add labels for the endpoints of the chords (as shown in the following diagram), you will see that $\Delta ABO \cong \Delta CDO$ because $OA = OC$, $OB = OD$, and the included angles are both right angles.

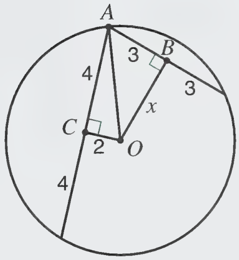
$$\therefore\ CD = AB$$

$$x = \frac{16}{2}$$

$$= 8$$



g. You can draw an equivalent diagram that makes solving for x simpler. Start with the diagram on the right.



The diagram has the two chords, each bisected by a perpendicular from the centre. In the diagram, two right triangles share a hypotenuse, AO: triangle ACO and triangle ABO.

Use the Pythagorean Theorem twice to find x.

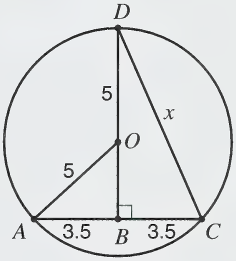
In triangle ACO,

$$\begin{aligned}(AO)^2 &= (OC)^2 + (AC)^2\\ (AO)^2 &= 2^2 + 4^2\\ (AO)^2 &= 4 + 16\\ (AO)^2 &= 20\end{aligned}$$

In triangle ABO,

$$\begin{aligned}(AO)^2 &= (OB)^2 + (AB)^2\\ 20 &= x^2 + 3^2\\ 20 &= x^2 + 9\\ x^2 &= 20 - 9\\ x^2 &= 11\\ x &= \sqrt{11}\\ &\doteq 3.32\end{aligned}$$

h. You can draw an equivalent diagram that makes solving for x simpler. Start with the diagram on the right.



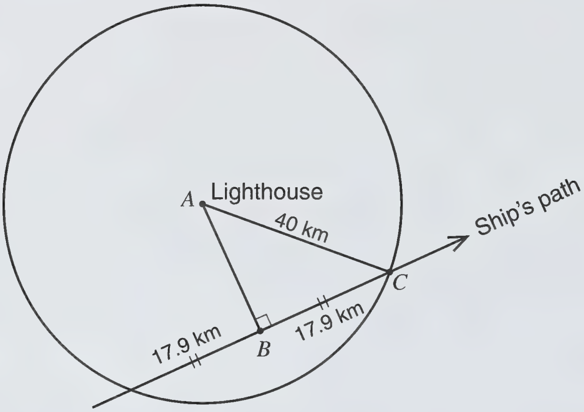
To apply the Pythagorean Theorem to the triangle containing the unknown x, you first have to find the lengths of the other two sides. Because chord AC is bisected, AB and BC each have a length of 3.5. The missing portion of the side passing through the centre, OB, can be found by applying the Pythagorean Theorem to triangle ABO.

$$\begin{aligned}(AO)^2 &= (AB)^2 + (OB)^2\\ 5^2 &= 3.5^2 + (OB)^2\\ 25 &= 12.25 + (OB)^2\\ (OB)^2 &= 25 - 12.25\\ (OB)^2 &= 12.75\\ OB &= \sqrt{12.75}\\ &\doteq 3.57\end{aligned}$$

$$\begin{aligned}\therefore\ (CD)^2 &= (BC)^2 + (DB)^2\\ x^2 &= 3.5^2 + (5 + OB)^2\\ x^2 &\doteq 12.25 + (5 + 3.57)^2\\ x^2 &\doteq 12.25 + 8.57^2\\ x^2 &\doteq 12.25 + 73.44\\ x^2 &\doteq 85.69\\ x &\doteq \sqrt{85.69}\\ &\doteq 9.26\end{aligned}$$

Activity 2 (continued)

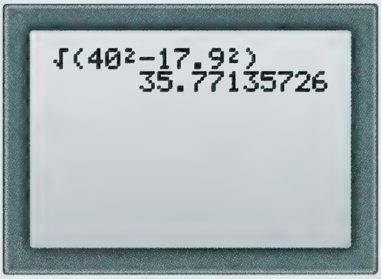
3. Draw the diagram to organize the information given in the question. The chord that represents the ship's path is 35.8 km long, and the distance from both endpoints of the chord to the lighthouse is 40 km.



The ship will be closest to the lighthouse when the line from the lighthouse to the ship is perpendicular to the path of the ship. That means segment *AB* will be a perpendicular bisector of the chord, making segment *BC* 17.9 km in length.

Apply the Pythagorean Theorem to find the distance from the ship to the lighthouse at its closest point.

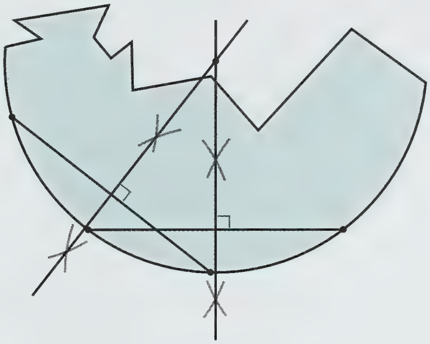
$$\begin{aligned}(AC)^2 &= (BC)^2 + (AB)^2\\ 40^2 &= 17.9^2 + (AB)^2\\ (AB)^2 &= 40^2 - 17.9^2\\ (AB)^2 &= 1600 - 320.41\\ (AB)^2 &= 1279.59\\ AB &= \sqrt{1279.59}\\ &\doteq 35.77\end{aligned}$$



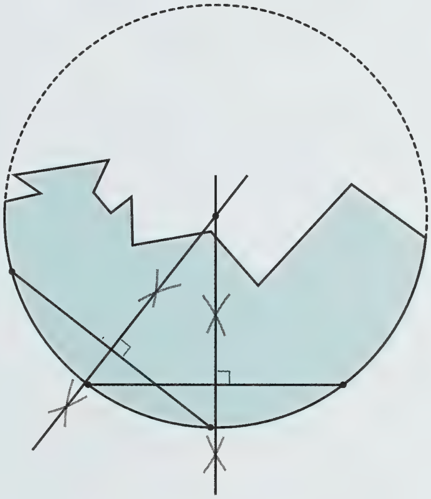
```
√(40²-17.9²)
         35.77135726
```

The closest the ship was to the lighthouse was about 35.77 km.

4. When the plate fragment is traced into your notebook, you can use perpendicular bisectors of chords to find the centre of the circle. Draw two chords on the sketch; then construct the perpendicular bisectors for both chords. The point where the two bisectors meet is the centre of the circular plate.

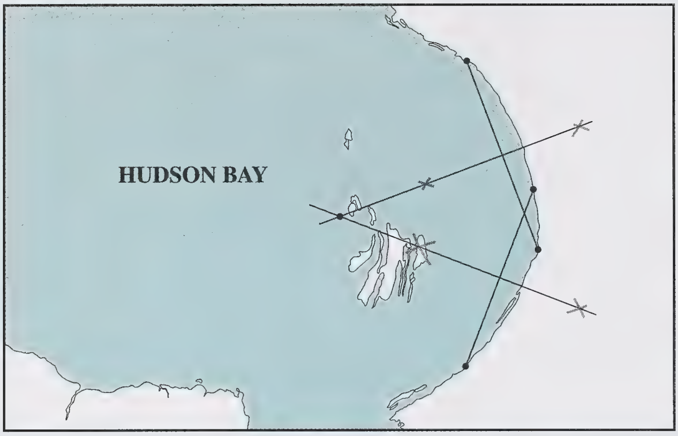


Complete the sketch using a compass.



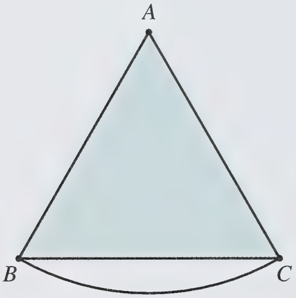
Activity 2 (continued)

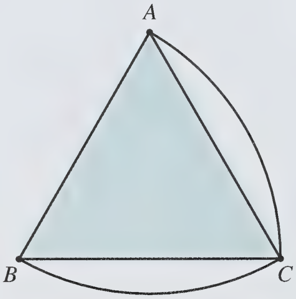
6. When the map is traced into your notebook, you can use perpendicular bisectors of chords to find the centre of the circle. Draw two chords on the sketch; then construct the perpendicular bisectors for both chords. The point where the two bisectors meet is the centre of the circular depression caused by the meteorite.

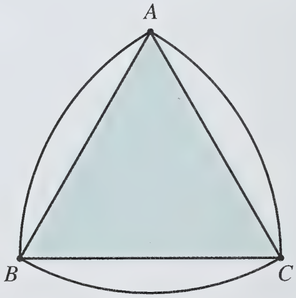


Finally, determine the distance from the shoreline to the centre of impact. The scale of this diagram is approximately $3 \text{ mm} = 25 \text{ km}$. Because the centre measures approximately 30 mm from the shoreline, the distance from the shore to the centre of impact is approximately 250 km.

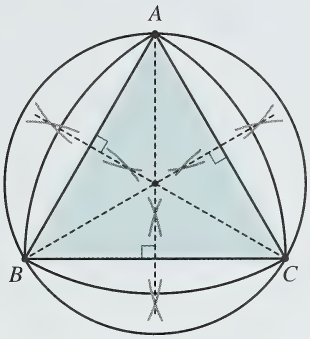
8. **a.**







b. The intersection of the three altitudes of the equilateral triangle is the centre of the circle.



c. Answers may vary.

An advantage of the Reuleaux triangle over a circle is that it takes less material to make it.

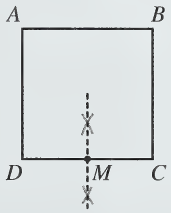
A disadvantage is that the construction is considerably more difficult. Another disadvantage is that the "points" make the shape more difficult to move since it will not roll easily. A third disadvantage is that a Reuleaux triangle utility cover would have to be rotated to be placed in one of three positions, whereas a round utility cover will fit without any rotation to a set position.

b. Textbook exercise 10 of "Exercises: Extending Your Thinking," p. 310

10.
$$\begin{aligned}\frac{1+\sqrt{5}}{2} &\doteq \frac{1+2.236}{2}\\ &\doteq \frac{3.236}{2}\\ &\doteq 1.618\end{aligned}$$

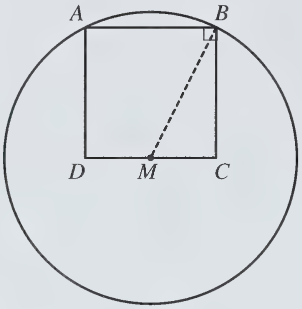
The ratio of the longer side to the shorter side is approximately 1.62:1. The construction of a golden rectangle is shown in the following sequence of diagrams.

Construct square *ABCD*; then construct the midpoint, *M*, of *CD*.

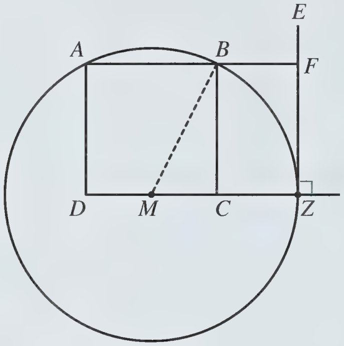


Activity 2 (continued)

Draw a circle with centre at *M* and radius *MB*.



Extend *DC* to intersect the circle at *Z*; construct a line *ZE* perpendicular to *DZ*; and extend *AB* to intersect *ZE* at *F*.



Find the ratio of *DZ* to *AD*.

$DZ = 32$ mm and $AD = 20$ mm

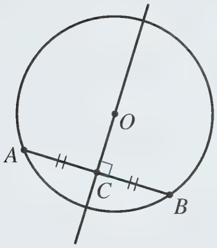
$\therefore\ DZ : AD$

$32 : 20$

$1.6 : 1$

The ratio of *DZ* to *AD* is approximately 1.6:1. This is close to the ratio calculated earlier.

11. Textbook exercise "Communicating the Ideas," p. 311



Two properties of chords of a circle were discovered:

- The perpendicular from the centre of a circle to a chord bisects the chord.
- The perpendicular bisector of any chord passes through the centre of the circle.

Activity 3: Properties of Angles in Circles

1. **a.** The measure of $\angle ABC$ is 65°.
   b. The measure of $\angle ABC$ is 180°.
   c. The measure of $\angle ABC$ is 122°.

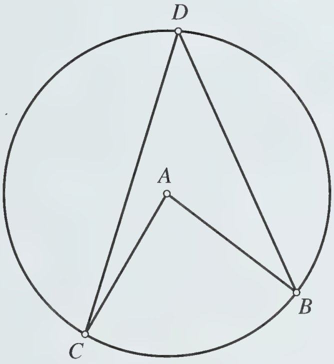
2. A right angle is an angle measuring 90°.
   An acute angle is an angle measuring less than 90°.
   An obtuse angle is an angle measuring greater than 90° but less than 180°.

3. **a. Textbook exercises 11 and 12 of "Investigation 1: Central Angle and Inscribed Angle," p. 314**

   11. Your diagram will look similar to the following. The central angle and inscribed angle measures will vary.

   $m\angle BAC = 82.8°$

   $m\angle BDC = 41.4°$



   The measure of $\angle BAC$ is twice the measure of $\angle BDC$.

   12. The measure of $\angle BAC$ will be twice the measure of $\angle BDC$.

   b. Textbook exercise 14 of "Investigation 1: Central Angle and Inscribed Angle," p. 314

   14. In any circle, the measure of the central angle is twice the measure of the inscribed angle subtended by the same arc.

Activity 3 (continued)

4. a. The measure of the central angle is twice the measure of the inscribed angle in each circle.

b. In any circle, the measure of the central angle is twice the measure of the inscribed angle subtended by the same arc.

5. Textbook exercises 1.a., 1.d., and 1.f. of "Exercises: Checking Your Skills," p. 319

1. a. Since the central angle is 120°, the inscribed angle, x, is half of 120°, or 60°.

d. Since the inscribed angle is 32°, the central angle, x, is twice 32°, or 64°.

f. Notice that a major arc subtends the inscribed angle. This does not change the relationship between the central angle and the inscribed angle; but be aware that the central angle is greater than 180°. Because the inscribed angle is 110°, the central angle, x, is twice 110°, or 220°.

6. a. Textbook exercises 9 and 10 of "Investigation 2: Angle in a Semicircle," pp. 315 and 316

9. The measure of an inscribed angle subtended by a half circle is 90°.

10. The measure of $\angle ADB$ will be 90°.

b. Textbook exercise 12 of "Investigation 2: Angle in a Semicircle," p. 316

12. In any circle, the measure of an inscribed angle subtended by a diameter is 90°.

7. a. The measure of each of the six angles subtended by the diameters is 90°.

b. In any circle, the measure of the inscribed angle subtended by a diameter is 90°.

8. Textbook exercise 1.b. of "Exercises: Checking Your Skills," p. 319

1. b. Because the inscribed angle is subtended by a diameter, $x = 90°$.

9. a. Textbook exercises 7 and 8 of "Investigation 3: Inscribed Angles," p. 317

7. The measures of $\angle BDC$ and $\angle BEC$ are the same.

8. For other positions of D and E and for other positions of B and C, the measures of $\angle BDC$ and $\angle BEC$ will be the same.

b. Textbook exercise 10 of “Investigation 3: Inscribed Angles,” p. 317

10. In any circle, inscribed angles subtended by the same arc are of equal measure.

10. a. The measure of the three angles in each circle are the same.

b. In any circle, inscribed angles subtended by the same arc are of equal measure.

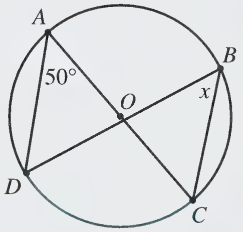
11. Textbook exercises 1.c., 1.e., 2, 3, 4, and 6 of “Exercises: Checking Your Skills, pp. 319 to 321

1. c. Because both inscribed angles are subtended by the same arc, $x = 35^\circ$.

e. Because the central angle is 60°, the measure of the inscribed angle, x, subtended by the same arc is half of 60°, or 30°.

2. a. Because the central angle subtended by the same arc is 130°, both x and y will be half of 130°, or 65°.

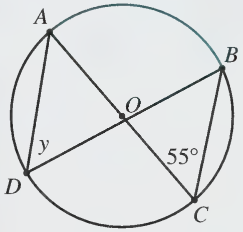
b. First, find the value of x.



Examine the labelled version of the diagram on the right. Note that $\angle DAC$ and $\angle DBC$ are subtended by the same arc, DC. This means they have the same measure.

Therefore, $x = 50^\circ$.

Now, find the value of y.

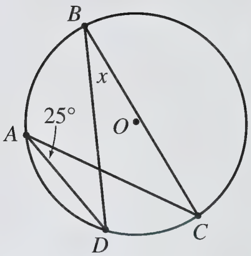


Examine the diagram on the right. Note that $\angle ADB$ and $\angle ACB$ are subtended by the same arc, AB. This means they have the same measure.

Therefore, $y = 55^\circ$.

Activity 3 (continued)

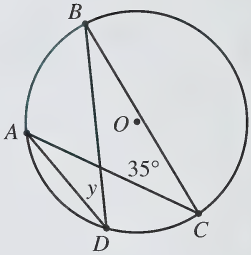
c. First, find the value of x.



Examine the diagram on the right. Note that both $\angle DAC$ and $\angle DBC$ are subtended by the same arc, DC. This means they have the same measure.

Therefore, $x = 25°$.

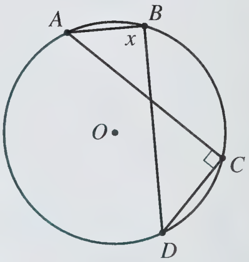
Now, find the value of y.



Examine the diagram on the right. Note that $\angle ADB$ and $\angle ACB$ are subtended by the same arc, AB. This means they have the same measure.

Therefore, $y = 35°$.

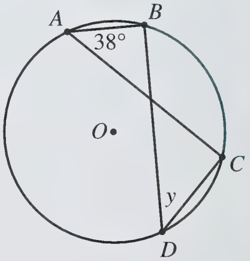
d. First, find the value of x.



Examine the diagram on the right. Note that $\angle ABD$ and $\angle ACD$ are subtended by the same arc, AD. This means they have the same measure.

Therefore, x is a right angle, or $x = 90°$.

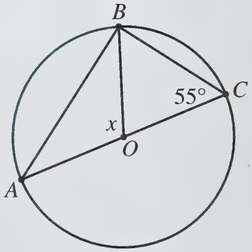
Now, find the value of y.



Examine the diagram on the right. Note that $\angle BAC$ and $\angle BDC$ are subtended by the same arc, BC. This means they have the same measure.

Therefore, $y = 38°$.

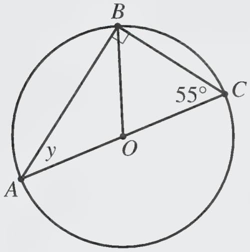
e. First find the value for x.



Examine the diagram on the right. Note that the inscribed angle $\angle BCA$ and the central angle $\angle BOA$ are subtended by the same arc, AB.

Therefore, x is twice 55°, or 110°.

Now, find the value of y.

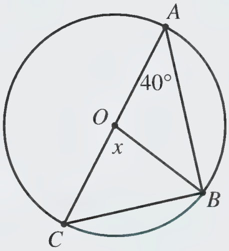


Examine the diagram on the right. Note that $\angle ABC$ is a right angle because it is subtended by a diameter of the circle, AOC.

Because the sum of the angles of a triangle is 180°,

$$\begin{aligned} y &= 180° - (90° + 55°) \\ &= 180° - 145° \\ &= 35° \end{aligned}$$

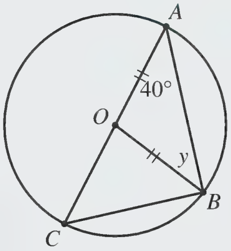
f. First, find the value of x.



Examine the diagram on the right. Note that the inscribed angle $\angle CAB$ and the central angle $\angle COB$ are subtended by the same arc, BC.

Therefore, x is twice 40°, or 80°.

Now, find the value of y.



Examine the diagram on the right. Note that $\angle ABO$ and $\angle BAO$ are base angles of an isosceles triangle with the equal sides being radii of the circle.

Therefore, $\angle ABO$ and $\angle BAO$ are equal, making $y = 40°$.

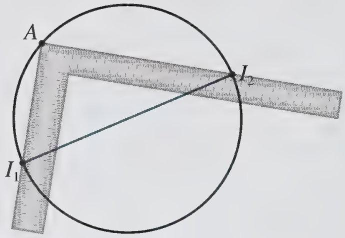
3. Answers will vary. Sample answers are given.

a. Players near the centre of the ice will be closer to the net and their shots will give the goalie less time to react. Players near the edge of the ice will have a smaller shooting angle and a further distance to shoot, thus having a harder time scoring.

b. If the net were a chord on a circle with the players arranged around the circumference of this circle, they would all have the same shooting angle and a more equal chance to score.

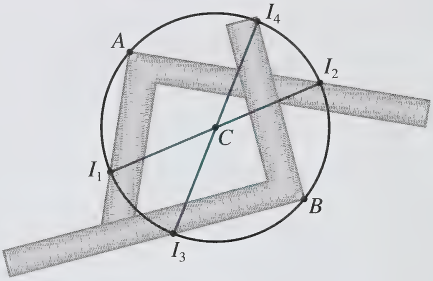
Activity 3 (continued)

4. Place the square on the circle so the right angle is on the circumference of the circle (like point *A* in the diagram). Then mark the points where the outside edge of the arms of the square meet the circumference of the circle (points I_1 and I_2 in the diagram). Because a right angle is subtended only by a diameter, the line joining I_1 and I_2 is a diameter of the circle.



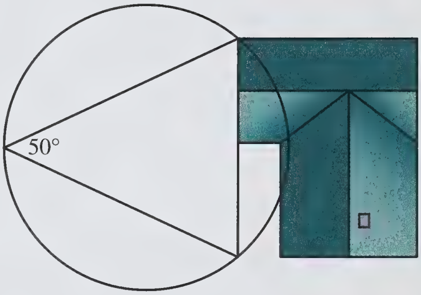
Place the square in a different place on the circle and follow the same process. (The diagram on the right shows this with point *B* and intersection points I_3 and I_4.) The line joining I_3 and I_4 is also a diameter of the circle.

The point where the two diameters meet, point *C*, is the centre of the circle.

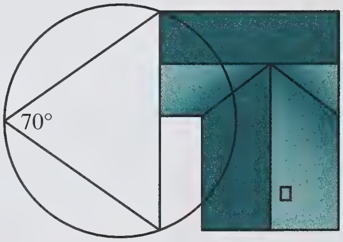


6. **a.** If the front of the house is considered to be a chord of a circle, the positions Lori can stand in become clear. Because the angle of view is 50°, any other position on the major arc of the circle will also have an angle of view of 50°. This is because every inscribed angle subtended by the same arc has the same measure.

Therefore, Lori can stand on any point on the major arc of the circle subtended by the house and get a picture that doesn't include the neighbours' properties.



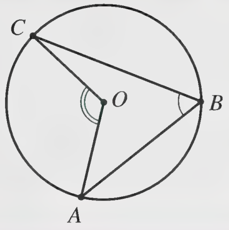
b. If the front of the house is considered to be a chord of a circle, the positions Gita can stand in become clear. Because the angle of view is 70°, any other position on the major arc of the circle will also have an angle of view of 70°. This is because every inscribed angle subtended by the same arc has the same measure.



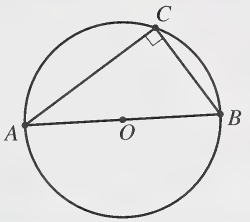
Therefore, Gita can stand on any point on the major arc of the circle subtended by the house and get a picture that doesn't include the neighbours' properties.

c. In both Lori's and Gita's cases, they can stand on an arc of a circle to take their pictures. Lori's circle will have a larger radius than Gita's circle. In other words, Lori will generally be farther from the house to get a picture showing the same features of the house.

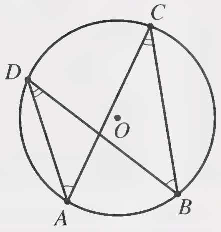
12. **Textbook exercise "Communicating the Ideas," p. 321**



The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.



An angle inscribed in a semicircle is a right angle.



Inscribed angles subtended by the same arc of a circle are equal.

Activity 4: Properites of Tangents to a Circle

1. **Textbook exercises 10, 11, and 13 of "Investigation 1: Angle Between a Tangent Line and Radius," p. 326**

    10. $\angle ABD$ is always a right angle.

    11. The angle between the tangent line and the radius is always a right angle, 90°.

    13. The measure of the angle between a tangent to a circle and the radius at the point of tangency is always 90° (a right angle).

2. **a.** The measure of the angle between the tangent line and the radius in each circle is 90°.

    b. The measure of the angle between a tangent to a circle and the radius at the point of tangency is always 90° (a right angle).

3. **Textbook exercises 8, 9, 10, and 12 of "Investigation 2: The Lengths of Tangent Segments from an External Point," pp. 327 and 328**

    8. The measures of *CE* and *ED* are equal.

    9. The measures of *CE* and *ED* are equal wherever point *C* or point *D* is moved around the circle.

    10. The measures of the lengths of the tangent segments will be the same for each circle.

    12. The lengths of the tangent segments to a circle from an external point are equal.

Activity 4 (continued)

4. **a.** The distances from point A to the points of tangency are the same for each circle.

   b. The lengths of the tangent segments to a circle from an external point are equal.

5. **Textbook exercises 11, 12, and 14 of "Investigation 3: The Angle Between a Tangent and a Chord and the Inscribed Angle on the Opposite Side of the Chord," pp. 329 and 330**

   11. The measures of $\angle EDB$ and $\angle EBF$ are equal.

   12. The measure of the angle between the chord and the tangent and the measure of the inscribed angle that is opposite the chord will be equal.

   14. The measure of the angle between a tangent and a chord is equal to the measure of the inscribed angle on the opposite side of the chord.

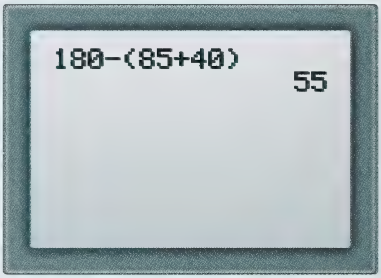
6. **a.** The measure of the inscribed angle and the measure of the angle between the tangent and the chord are equal in each diagram.

   b. The measure of the angle between a tangent and a chord is equal to the measure of the inscribed angle on the opposite side of the chord.

7. **Textbook exercises 1 and 2 of "Practise Your Prior Skills," p. 333**

   1. **a.** The sum of the angles in a triangle is 180°.

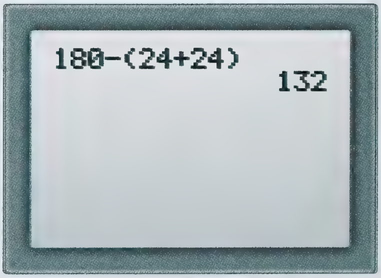
$$
\begin{aligned}
\therefore\ 180^\circ &= 85^\circ + 40^\circ + x \\
180^\circ &= 125^\circ + x \\
x &= 180^\circ - 125^\circ \\
&= 55^\circ
\end{aligned}
$$



```
180-(85+40)
                55
```

   b. Because the triangle is isosceles, the angles opposite the equal sides both measure 24°.

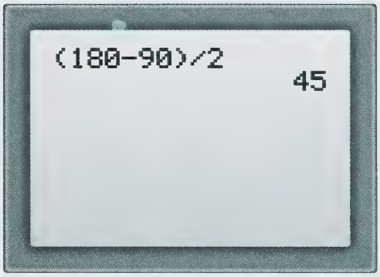
$$
\begin{aligned}
\therefore\ 180^\circ &= 24^\circ + 24^\circ + x \\
180^\circ &= 48^\circ + x \\
x &= 180^\circ - 48^\circ \\
&= 132^\circ
\end{aligned}
$$



```
180-(24+24)
               132
```

c. Because the triangle is isosceles, the angles opposite the equal sides are both equal to x.

$$\begin{aligned} \therefore\ 180^\circ &= 90^\circ + x^\circ + x \\ 180^\circ &= 90^\circ + 2x \\ 2x &= 180^\circ - 90^\circ \\ 2x &= 90^\circ \\ x &= 45^\circ \end{aligned}$$



```
(180-90)/2
                45
```

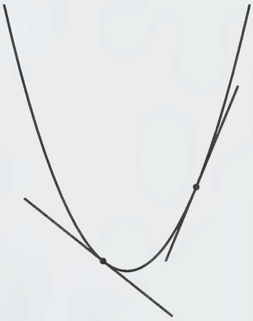
d. Vertically opposite angles are equal. Therefore, $x = 52^\circ$.

2. **a.** The triangles are congruent because two sides and the included angle are of equal measure.

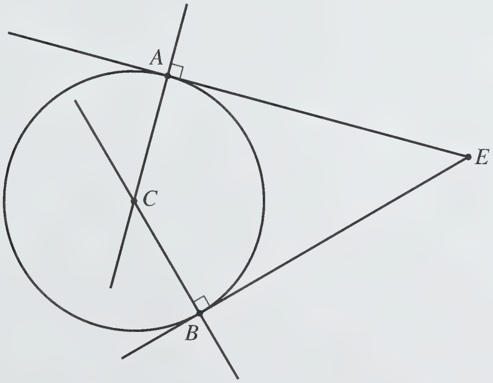
b. The triangles are congruent because two angles and the included side are of equal measure.

8. Textbook exercises 1 to 4 of "Discussing the Ideas," p. 332

1. Yes, a tangent to a parabola could be described as a line that intersects the parabola at exactly one point. The diagram on the right shows two such tangent lines.

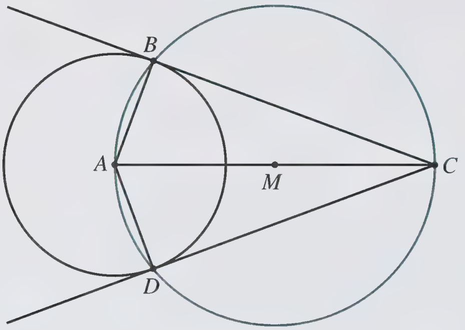


2. Because a radius to a point of tangency is perpendicular to the tangent, draw a perpendicular to the tangent at the point of tangency. This perpendicular must pass through the centre of the circle. If you do this for both tangents, the point where the perpendiculars meet is the centre of the circle.



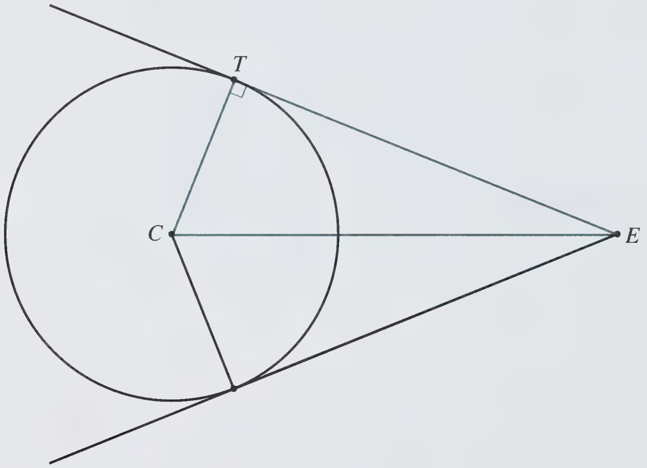
Activity 4 (continued)

3. The centre of the circle will be the midpoint of the line segment joining *A* and *C*.



4. Because the radius is at a right angle to the tangent segment, the triangle with vertices at centre *C*, point of tangency *T*, and external point *E* is a right triangle.

   The Pythagorean Theorem applies only to right triangles; therefore, you can use the Pythagorean Theorem to calculate the lengths of the tangent segments.

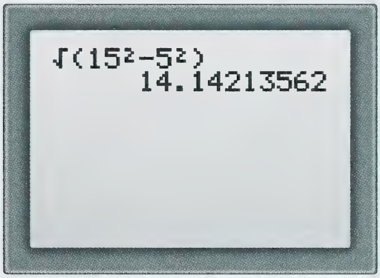


9. Textbook exercises 1.a., 1.b., 1.c., 1.f., 2, 3.a., 3.c., 3.e., 3.f., and 6 of "Exercises: Checking Your Skills," pp. 333 to 336

1. **a.** Because *OB* is a radius and *AB* is tangent to the circle, triangle *ABO* is a right triangle.

   Apply the Pythagorean Theorem to find x.

$$\begin{aligned}(OA)^2 &= (OB)^2 + (AB)^2\\ 15^2 &= 5^2 + x^2\\ 225 &= 25 + x^2\\ x^2 &= 225 - 25\\ x^2 &= 200\\ x &= \sqrt{200}\\ &\doteq 14.14\end{aligned}$$



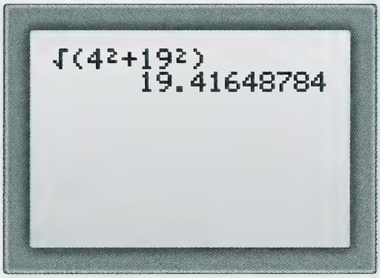
```
√(15²-5²)
         14.14213562
```

   Because the tangent segments from an external point are equal, $y \doteq 14.14$.

   b. Because *AB* and *AC* are tangent segments from point *A*, they are equal. Therefore, $y = 19$.

   To find x, apply the Pythagorean Theorem to triangle *ABO*.

$$\begin{aligned}(OA)^2 &= (OB)^2 + (AB)^2\\ x^2 &= 4^2 + 19^2\\ x^2 &= 16 + 361\\ x^2 &= 377\\ x &= \sqrt{377}\\ &\doteq 19.42\end{aligned}$$



```
√(4²+19²)
         19.41648784
```

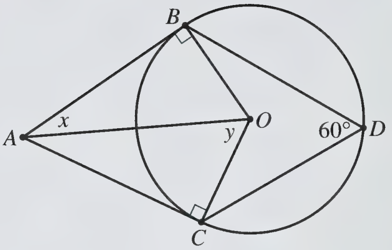
   c. Because tangent segments from an external point (in this case *P*) are equal, $y = 7$.

   Because tangent segments from an external point (in this case *R*) are equal, $x = 12$.

   f. The angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord. Because x and y are inscribed angles subtended by chord *CD*, $x = y = 75°$.

Activity 4 (continued)

2. **a.** The central angle, $\angle BOC$, is twice the inscribed angle, $\angle BDC$. Therefore, $\angle BOC = 120°$.

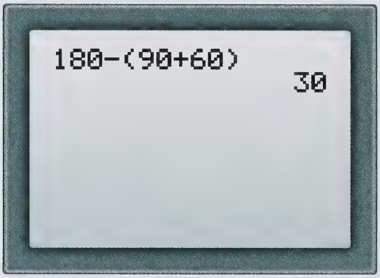


Triangle *ABO* and triangle *ACO* are congruent because $AB = AC$, $BO = CO$, and side *AO* is common. Therefore, $\angle BOA = \angle COA$.

Because $\angle BOA = \angle COA$, *y* must be half of 120°. Therefore, $y = 60°$.

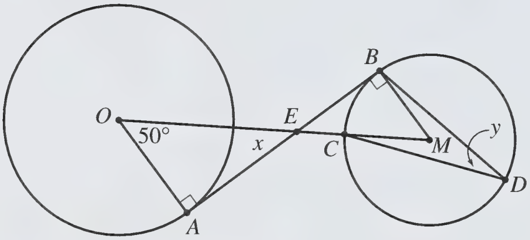
$\angle ABO = 90°$, since *AB* is a tangent and *BO* is a radius at the point of tangency, and $\angle BOA = 60°$. The sum of angles of a triangle is 180°; so, use this information to calculate the value for *x*.

$$\begin{aligned} 180° &= \angle ABO + \angle AOB + \angle BAO \\ 180° &= 90° + 60° + x \\ 180 &= 150° + x \\ x &= 180° - 150° \\ &= 30° \end{aligned}$$



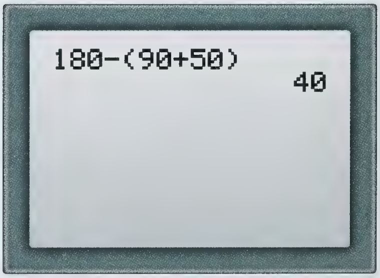
```
180-(90+60)
                30
```

b.



The sum of the angles in triangle *OAE* is 180°.

$$\begin{aligned} \therefore\ 180° &= \angle EAO + \angle AOE + \angle AEO \\ 180° &= 90° + 50° + x \\ 180 &= 140° + x \\ x &= 180° - 140° \\ &= 40° \end{aligned}$$

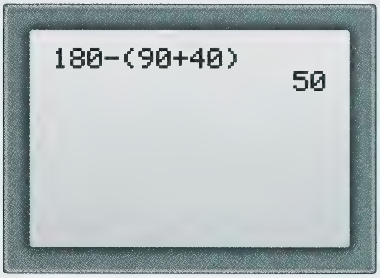


```
180-(90+50)
                40
```

Because $\angle BEM$ and $\angle AEO$ are vertically opposite angles, $\angle BEM = 40°$.

The sum of the angles in triangle *MBE* is 180°; therefore, find the measure of the central angle, $\angle BME$.

$$\begin{aligned} 180^\circ &= \angle EBM + \angle BEM + \angle BME \\ 180^\circ &= 90^\circ + 40^\circ + \angle BME \\ 180^\circ &= 130^\circ + \angle BME \\ \angle BME &= 180^\circ - 130^\circ \\ &= 50^\circ \end{aligned}$$



```
180-(90+40)
             50
```

The central angle $\angle BMC$, which is the same as $\angle BME$, is twice the inscribed angle $\angle BDC$.

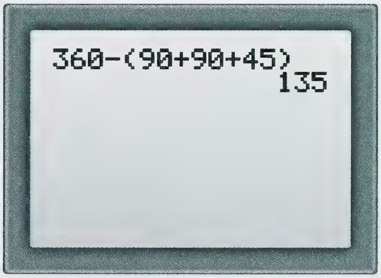
$$\begin{aligned} \therefore\ 2(\angle BDC) &= \angle BMC \\ 2y &= 50^\circ \\ y &= \frac{50^\circ}{2} \\ &= 25^\circ \end{aligned}$$

3. **a.** *PB* is tangent to the circle at *B*, and *OB* is a radius of the circle. Therefore, $x = 90^\circ$.

Since *PA* is tangent to the circle at *A*, $\angle PAO = 90^\circ$.

Recall that the sum of the angles in a quadrilateral is 360°.

$$\begin{aligned} \therefore\ 360 &= 90^\circ + 90^\circ + 45^\circ + y \\ 360^\circ &= 225^\circ + y \\ y &= 360^\circ - 225^\circ \\ &= 135^\circ \end{aligned}$$



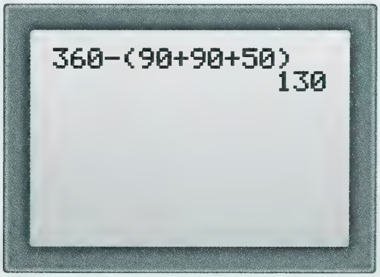
```
360-(90+90+45)
           135
```

c. $\angle APB$ is vertically opposite a 50° angle. Therefore $x = 50$.

PA is tangent to *AO* and *PB* is tangent to *BO*; so, $\angle PAO = 90^\circ$ and $\angle PBO = 90^\circ$.

Recall that the sum of the angles of a quadrilateral is 360°.

$$\begin{aligned} \therefore\ 360 &= 90^\circ + 90^\circ + 50^\circ + y \\ 360^\circ &= 230^\circ + y \\ y &= 360^\circ - 230^\circ \\ &= 130^\circ \end{aligned}$$



```
360-(90+90+50)
           130
```

e. $\angle BOA = 360° - 265°$

$= 95°$

Because x is an inscribed angle subtended by the same arc as the central angle $\angle BOA$,

$$\begin{aligned} 2x &= \angle BOA \\ 2x &= 95° \\ x &= \frac{95°}{2} \\ &= 47.5° \end{aligned}$$

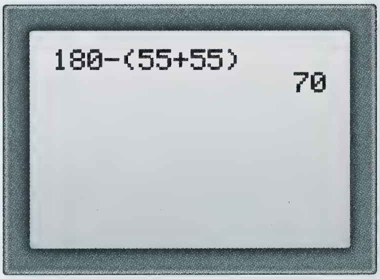
Both x and y have the same value because x is the measure of an inscribed angle opposite a chord (in this case AB) and y is the measure of an angle between the same chord and a tangent line. Therefore, $y = 47.5°$.

f. Because $\angle CAB$ is an inscribed angle opposite chord CB and the measure of the angle between the same chord and a tangent is 55°, the measure of $\angle CAB$ is 55°.

Because $CA = CB$, $\angle ABC = \angle CAB = 55°$.

Calculate the value of y.

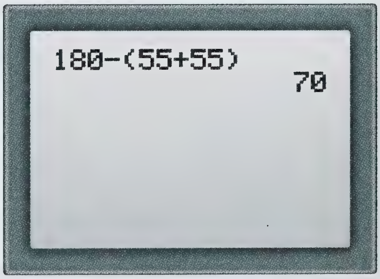
$$\begin{aligned} 180° &= \angle ACB + \angle CAB + \angle ABC \\ 180° &= y + 55° + 55° \\ 180° &= y + 110° \\ y &= 180° - 110° \\ &= 70° \end{aligned}$$



```
180-(55+55)
                70
```

Because a straight angle is 180°, $\angle ABP$ can be calculated as follows:

$$\begin{aligned} 180° &= 55° + \angle ABC + \angle ABP \\ 180° &= 55° + 55° + \angle ABP \\ 180° &= 110° + \angle ABP \\ \angle ABP &= 180° - 110° \\ &= 70° \end{aligned}$$

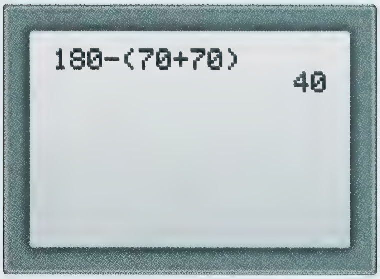


```
180-(55+55)
                70
```

You could have also used the following property of a tangent to a circle to determine the measure of $\angle ABP$: *the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord.*

Because $AP = BP$, $\angle BAP = \angle ABP = 70°$.

$$\begin{aligned} \therefore\ 180° &= \angle APB + \angle BAP + \angle ABP \\ 180° &= x + 70° + 70° \\ 180° &= x + 140° \\ x &= 180° - 140° \\ &= 40° \end{aligned}$$



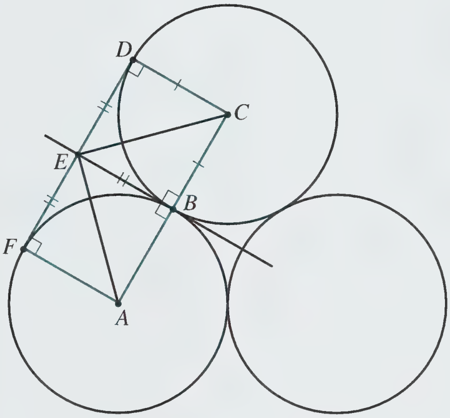
```
180-(70+70)
                40
```

6. This problem becomes simpler to answer if it is broken into several smaller parts. You should look at it as six separate parts: three of them are the curved parts around the logs and the other three are the straight parts tangent to the logs.

The curved parts are each one-third of the circumference of a log, $120° = \frac{1}{3} \times 360°$. Therefore, the amount of strapping around each curved part is $\frac{1}{3}\pi d$. (Recall that the circumference of a circle is πd.)

Because the strapping goes around three logs, the amount of strapping around the curved parts is

$$\begin{aligned} 3 \times \left(\frac{1}{3}\pi d\right) &= \cancel{3} \times \left(\frac{1}{\cancel{3}}\right)\pi(1.2) \\ &= 1.2\pi \\ &\doteq 3.77 \text{ m} \end{aligned}$$



Because $DF = AC$ and AC is equal to two radii or a diameter, each straight part is equal to a diameter or 1.2 m.

Therefore, the amount of strapping in the straight parts is $3 \times 1.2 = 3.6$ m.

You will need $3.6 \text{ m} + 3.77 \text{ m} \doteq 7.37 \text{ m}$ of strapping to go around the logs. The strapping should be cut longer, though, to fasten the strapping together.

10. Textbook exercise 9 of "Exercises: Extending Your Thinking," p. 337

9. You should break this problem into four parts: the length of the belt touching the large wheel, the length of the belt touching the small wheel, the length of the belt below the wheels, and the length of the belt above the wheels.

The lengths of the belt touching both wheels is as follows:

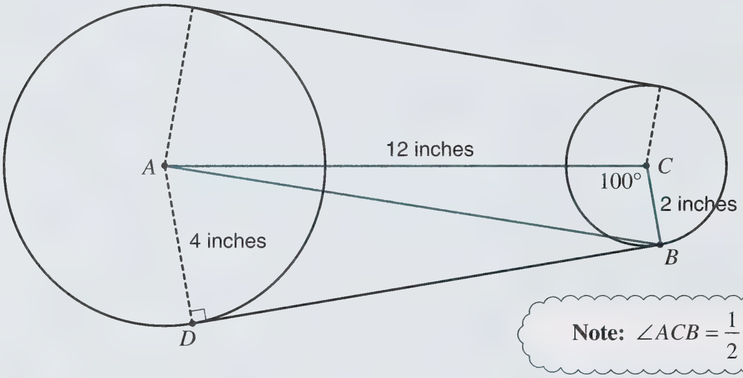
Length Around Large Wheel

$$\left(\frac{200^\circ}{360^\circ}\right)2\pi r=\left(\frac{5}{9}\right)2\pi(4)$$
$$=\frac{40}{9}\pi$$
$$\doteq 13.963 \text{ inches}$$

Length Around Small Wheel

$$\left(\frac{160^\circ}{360^\circ}\right)2\pi r=\left(\frac{4}{9}\right)2\pi(2)$$
$$=\frac{16}{9}\pi$$
$$\doteq 5.585 \text{ inches}$$

Use the following diagram to see how to arrrive at the calculations for the lengths of the upper and lower belts.

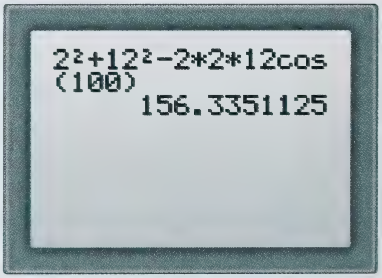


Note: $\angle ACB=\frac{1}{2}(360-160)$

First, use the Law of Cosines, which you learned in grade 10, to find the length of *AB*.

$$c^2=a^2+b^2-2ab\cos C$$
$$(AB)^2=2^2+12^2-2(2)(12)\cos 100^\circ$$
$$(AB)^2\doteq 156.335$$

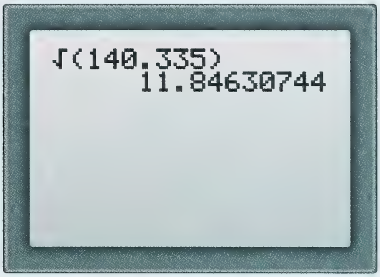
Note: Make sure your calculator is in degree mode.



```
2²+12²-2*2*12cos
(100)
         156.3351125
```

Now, use the Pythagorean Theorem to find the length of the lower belt, *BD*.

$$\begin{aligned}
(AB)^2 &= (AD)^2 + (DB)^2 \\
156.335 &\doteq 4^2 + (DB)^2 \\
156.335 &\doteq 16 + (DB)^2 \\
(DB)^2 &\doteq 156.335 - 16 \\
(DB)^2 &\doteq 140.335 \\
DB &\doteq 11.846
\end{aligned}$$

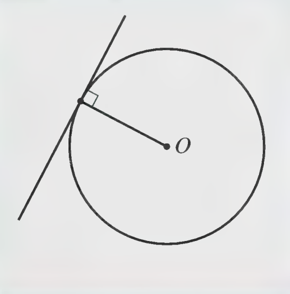


```
√(140.335)
         11.84630744
```

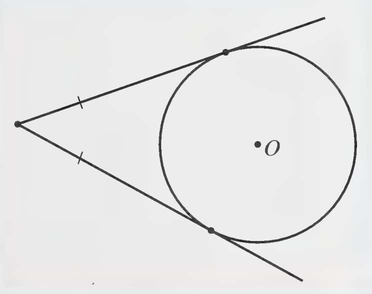
The length of the upper belt is the same as the length of the lower belt.

$$\begin{aligned}
\therefore\ \text{Total length of belt} &\doteq 13.963 + 5.585 + 11.846 + 11.846 \\
&\doteq 43.24\ \text{inches}
\end{aligned}$$

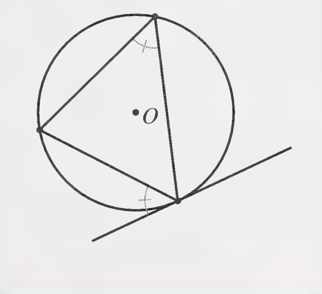
11. Textbook exercise "Communicating the Ideas," p. 337



A tangent to a circle is perpendicular to the radius at the point of tangency



The tangent segments to a circle from an external point are equal.

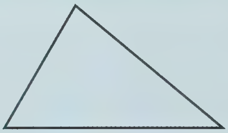
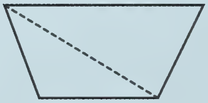
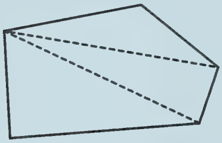
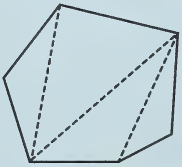
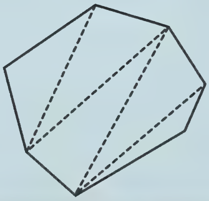


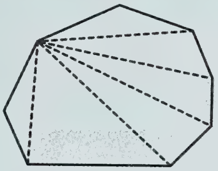
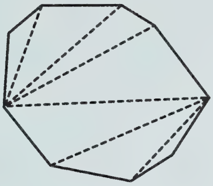
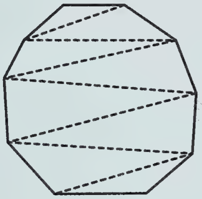
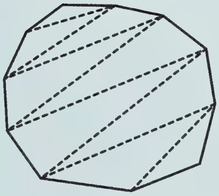
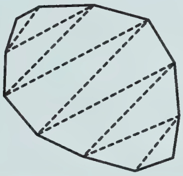
The angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord.

Activity 5: Properties of Polygons

1. **Textbook exercises 1 to 3 of "Investigation 1: Interior Angles in a Polygon," pp. 338 and 339**

1. a.

Number of Sides in a Polygon	Sketch	Number of Triangles Formed	Sum of the Interior Angles of the Polygon
3		1	$(1)(180^\circ) = 180^\circ$
4		2	$(2)(180^\circ) = 360^\circ$
5		3	$(3)(180^\circ) = 540^\circ$
6		4	$(4)(180^\circ) = 720^\circ$
7		5	$(5)(180^\circ) = 900^\circ$

8		6	$(6)(180°) = 1080°$
9		7	$(7)(180°) = 1260°$
10		8	$(8)(180°) = 1440°$
11		9	$(9)(180°) = 1620°$
12		10	$(10)(180°) = 1800°$

b. A 17-sided polygon will form 15 triangles. Therefore, the sum of the interior angles of a 17-sided polygon is $(15)(180°) = 2700°$.

c. An n-sided polygon will form $n-2$ triangles. Therefore, the sum of the interior angles of any polygon is $180°(n-2)$.

Activity 5 (continued)

2. **a.** Let S represent the sum of the interior angles.

$$\begin{aligned} \therefore\ S &= 180^\circ(n-2) \\ &= 90^\circ(2)(n-2) \\ &= 90^\circ(2n-4) \end{aligned}$$

Formula contains 90° rather than 180°.

b.

$$\begin{aligned} S &= 90^\circ(2n-4) \\ &= 180^\circ n - 360^\circ \end{aligned}$$

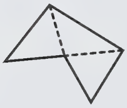
c. Answers may vary. Examples of other formulas are $45^\circ(4n-8)$, $22.5^\circ(8n-16)$, and $360^\circ\left(\frac{1}{2}n-1\right)$.

3. **a.** The sum of the interior angles is 360°.

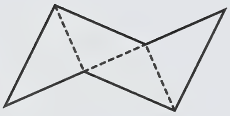
b.

$$\begin{aligned} 180^\circ(n-2) &= 180^\circ(4-2) \\ &= 180^\circ(2) \\ &= 360^\circ \end{aligned}$$

c. Diagrams will vary.



$$\begin{aligned} 180^\circ(n-2) &= 180^\circ(5-2) \\ &= 180^\circ(3) \\ &= 540^\circ \end{aligned}$$



$$\begin{aligned} 180^\circ(n-2) &= 180^\circ(6-2) \\ &= 180^\circ(4) \\ &= 720^\circ \end{aligned}$$

d. The values for the sum of the interior angles are the same whether the formula or the triangle method is used.

e. The formula from exercise 1.c. works for both convex and concave polygons. You can always find a way to break up the polygon into non-overlapping triangles.

2. **Textbook exercises 5, 6, 7, and 9 of "Investigation 2: Cyclic Quadrilaterals," p. 340**

    5. $\angle CDE$ and $\angle CBE$ add up to 180°.

    6. $\angle DCB$ and $\angle DEB$ also add up to 180°.

    7. The sum of the measures of each pair of opposite angles in a cyclic quadrilateral will be 180°.

    9. In any cyclic quadrilateral, the sum of the measures of each pair of opposite angles is 180°.

3. Diagrams will vary.

    a. The sum of each pair of opposite angles is 180°.

    b. The sum of the opposite pairs of angles in other cyclic quadrilaterals will be 180°. Yes, this prediction is valid.

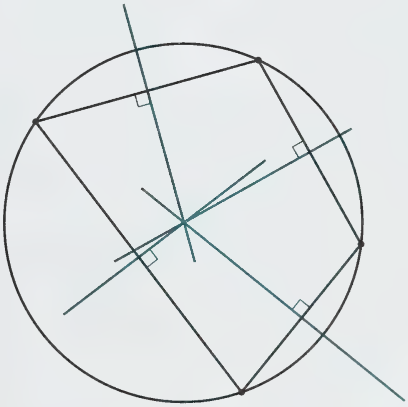
    c. In any cyclic quadrilateral, the sum of the measures of each pair of opposite angles is 180°.

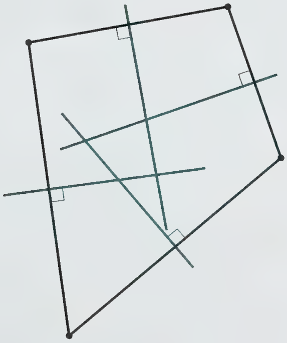
4. **Textbook questions 1 to 4 of "Discussing the Ideas," p. 342**

    1. The quadrilateral is either a square or a rectangle.

    2. In some cases, you will be able to construct a circle through four non-collinear points in a plane. Generally, though, only three of four non-collinear points will fit on a circle. If the perpendicular bisectors of the segments through pairs of the points all intersect at the same point, this will be the centre of the desired circle.

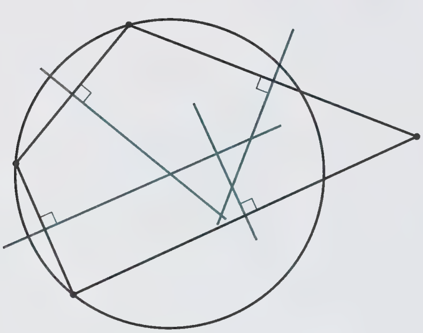
    If they all do not intersect at the same point, there will be no such circle.

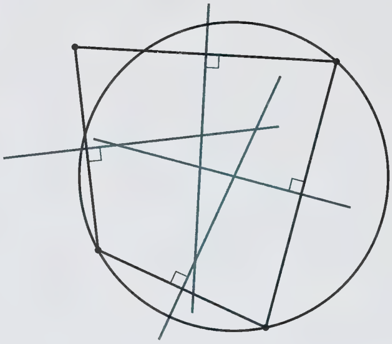




Activity 5 (continued)

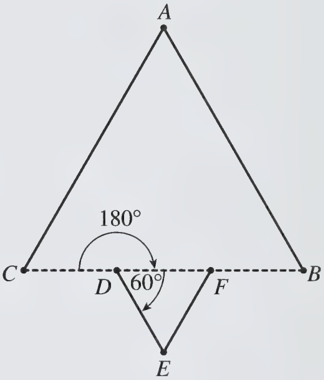
3. Diagrams will vary. Two examples of quadrilaterals that are not cyclic are given.





4. a. Look at the diagram on the right.

Triangle *ABC* is an equilateral triangle. Triangle *DEF* is added to side *CB*. $\angle CDB$ is a straight angle, or 180°; and $\angle FDE$ is 60°. Therefore, $\angle CDE$ is $180° + 60° = 240°$.



b. As additional triangles are added to the snowflake, they are added to the middle of a current side. This makes the angle calculations exactly the same as for those in the second polygon.

5. Textbook exercises 1 to 5 of "Exercises: Checking Your Skills," pp. 343 and 344

1. a. This is a six-sided polygon. Therefore, the sum of the angles is

$$\begin{aligned}180°(n-2) &= 180°(6-2)\\ &= 180°(4)\\ &= 720°\end{aligned}$$

b. This is a 12-sided polygon. Therefore, the sum of the angles is

$$\begin{aligned} 180^\circ(n-2) &= 180^\circ(12-2) \\ &= 180^\circ(10) \\ &= 1800^\circ \end{aligned}$$

c. This is a six-sided polygon. Therefore, the sum of the angles is 720°.

d. This is an eight-sided polygon. Therefore, the sum of the angles is

$$\begin{aligned} 180^\circ(n-2) &= 180^\circ(8-2) \\ &= 180^\circ(6) \\ &= 1080^\circ \end{aligned}$$

2. a. Opposite angles in a cyclic quadrilateral are supplementary.

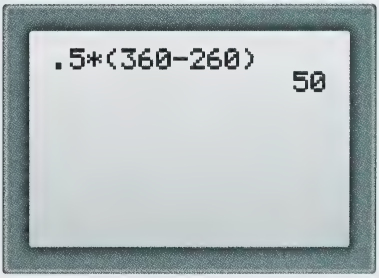
$$\begin{aligned} \therefore\ x+100^\circ &= 180^\circ \qquad \text{and} \qquad & y+110^\circ &= 180^\circ \\ x &= 80^\circ & y &= 70^\circ \end{aligned}$$

b. Opposite angles in a cyclic quadrilateral are supplementary.

$$\begin{aligned} \therefore\ x+90^\circ &= 180^\circ \qquad \text{and} \qquad & y+75^\circ &= 180^\circ \\ x &= 90^\circ & y &= 105^\circ \end{aligned}$$

c. First, find y. Recall that an inscribed angle is half the measure of the central angle subtended by the same arc. Notice that the central angle is the acute angle at the centre of the circle, not the obtuse angle.

$$\begin{aligned} \therefore\ y &= \frac{1}{2}(360^\circ - 260^\circ) \\ &= \frac{1}{2}(100^\circ) \\ &= 50^\circ \end{aligned}$$



```
.5*(360-260)
             50
```

Now, find x by applying the property of opposite angles in a cyclic quadrilateral. Notice that $\angle y$ and $\angle(x+70^\circ)$ are opposite angles of a cyclic quadrilateral.

$$\begin{aligned} \therefore\ (x+70^\circ)+y &= 180^\circ \\ x+70^\circ+50^\circ &= 180^\circ \\ x+120^\circ &= 180^\circ \\ x &= 180^\circ - 120^\circ \\ &= 60^\circ \end{aligned}$$

d. First, find x. Recall that opposite angles in a cyclic quadrilateral are supplementary.

$$\begin{aligned}\therefore\ x+(180^\circ-100^\circ)&=180^\circ\\ x+80^\circ&=180^\circ\\ x&=180^\circ-80^\circ\\ &=100^\circ\end{aligned}$$

Now find y. Notice that y is the supplement of one of two equal angles in an isosceles triangle. The third angle of the isosceles triangle is x.

Because the sum of the angles of a triangle is 180°,

$$\begin{aligned}x+2(180^\circ-y)&=180^\circ\\ 100^\circ+2(180^\circ-y)&=180^\circ\\ 100^\circ+360^\circ-2y&=180^\circ\\ 460^\circ-2y&=180^\circ\\ -2y&=180^\circ-460^\circ\\ -2y&=-280^\circ\\ y&=140^\circ\end{aligned}$$

3. The dam has a cross-sectional shape of a four-sided polygon. This quadrilateral will have a sum of interior angles of 360°. The measures of two of the angles are given directly: $\angle C=140^\circ$ and $\angle D=120^\circ$.

$\angle CBE$ is supplementary to $\angle ABC$.

$$\begin{aligned}\therefore\ \angle CBE&=180^\circ-\angle ABC\\ &=180^\circ-130^\circ\\ &=50^\circ\end{aligned}$$

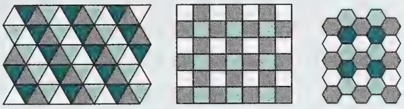
Now, find $\angle DEB$. Recall that the sum of the angles of a four-sided polygon is 360°.

$$\begin{aligned}\therefore\ 360^\circ&=\angle CBE+\angle C+\angle D+\angle DEB\\ 360^\circ&=50^\circ+140^\circ+120^\circ+\angle DEB\\ 360^\circ&=310^\circ+\angle DEB\\ \angle DEB&=360^\circ-310^\circ\\ &=50^\circ\end{aligned}$$

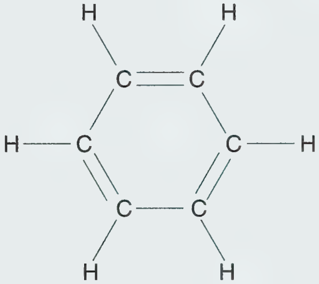
$\angle DEF$ is supplementary to $\angle DEB$.

$$\begin{aligned}\therefore\ \angle DEF &= 180^\circ - \angle DEB\\ &= 180^\circ - 50^\circ\\ &= 130^\circ\end{aligned}$$

4. You can use equilateral triangles, squares, or hexagons to tessellate a plane.



5. The carbon atoms form a hexagon. The sum of the angles of a hexagon is

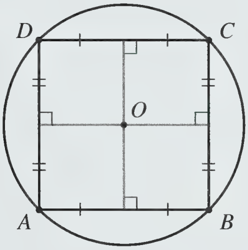


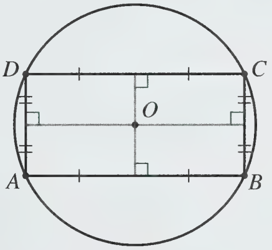
$$\begin{aligned}180^\circ(n-2) &= 180^\circ(6-2)\\ &= 180^\circ(4)\\ &= 720^\circ\end{aligned}$$

Because there are six equal angles, they will each be $\frac{1}{6}$ of 720°. Therefore, the angle formed by two segments that join three consecutive carbon nuclei is 120°.

6. **Textbook exercise "Communicating the Ideas," p. 345**

The square and the rectangle are always cyclic quadrilaterals because the perpendicular bisectors of their sides meet at one point.





In square *ABCD*, the perpendicular bisectors of the sides *AB*, *BC*, *CD*, and *AD* meet at point *O*. This point becomes the centre of the circumscribing circle.

In rectangle *ABCD*, the perpendicular bisectors of the sides *AB*, *BC*, *CD*, and *AD* meet at point *O*. This point becomes the centre of the circumscribing circle.

Follow-up Activities

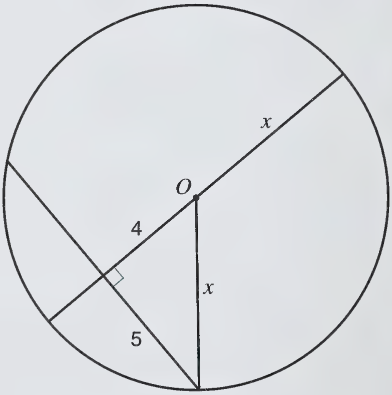
Textbook exercises 2.a., 2.c., 3, 4, 5.c., 6.a., 6.d., 7.b., 7.d., 8, 11, and 12 of Part B of "What Should I Be Able to Do?," pp. 349 to 352

2. a. Since a perpendicular from the centre of a circle to a chord bisects the chord, the right triangle has sides of length 13, 5 and $\frac{x}{2}$.

Apply the Pythagorean Theorem to find the value of *x*.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 13^2 &= 5^2 + \left(\frac{x}{2}\right)^2 \\ 169 &= 25 + \frac{x^2}{4} \\ \frac{x^2}{4} &= 169 - 25 \\ \frac{x^2}{4} &= 144 \\ x^2 &= 576 \\ x &= \sqrt{576} \\ &= 24 \end{aligned}$$

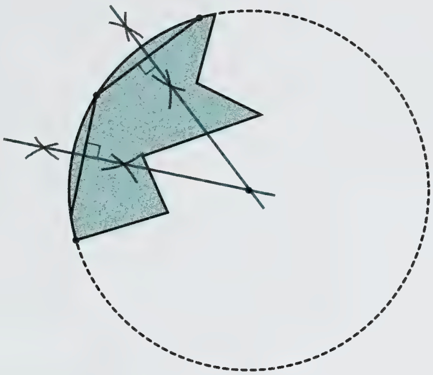
c. Redraw the diagram and insert a radius, *x*, that connects the centre with one of the endpoints of the chord. This makes a right triangle with side lengths measuring 4, 5, and *x*.



Apply the Pythagorean Theorem to find the value of *x*.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= 4^2 + 5^2 \\ x^2 &= 16 + 25 \\ x^2 &= 41 \\ x &= \sqrt{41} \\ &\doteq 6.4 \end{aligned}$$

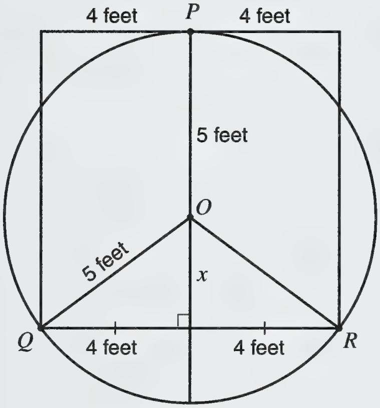
3. Draw two chords on the curved part of the fragment; then draw the perpendicular bisectors of these chords. The point where these bisectors meet is the centre of the circular object.



Measure the distance from the centre of the object to the edge of the object. This value is the radius.

The radius of the fragment is 2.8 cm or 28 mm.

4. Add the information given to the diagram. Refer to the new diagram given on the right. Let x represent the distance (in feet) from O to QR.



a. Apply the Pythagorean Theorem to find the value of x.

$$\begin{aligned}
c^2 &= a^2 + b^2 \\
5^2 &= 4^2 + x^2 \\
25 &= 16 + x^2 \\
x^2 &= 25 - 16 \\
x^2 &= 9 \\
x &= \sqrt{9} \\
&= 3
\end{aligned}$$

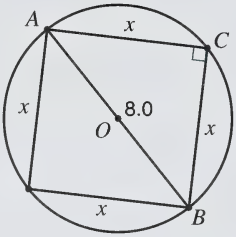
The distance from O to QR is 3 feet.

b. The height of the room is $5+3=8$ feet.

Follow-up Activities (continued)

5. **c.** The diagonal of the given square will have a length of 8.0. Because the diagonal forms a right triangle, $\angle ACB$, apply the Pythagorean Theorem to solve for *x*.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ 8^2 &= x^2 + x^2 \\ 64 &= 2x^2 \\ 2x^2 &= 64 \\ x^2 &= 32 \\ x &= \sqrt{32} \\ &\doteq 5.7 \end{aligned}$$



6. **a.** Because an inscribed angle subtended by a diameter is a right angle, both *x* and *y* are 90°.

d. Both *x* and *y* are subtended by the same arc as the central angle. Therefore, *x* and *y* are both half of 102°, or 51°.

7. **b.** The quadrilateral is a cyclic quadrilateral; thus, each pair of opposite angles are supplementary.

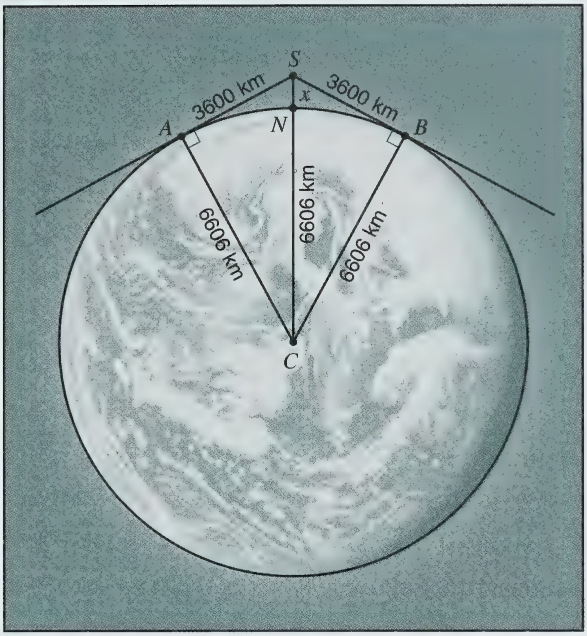
$$\begin{aligned} \therefore\ x + 130° &= 180° \\ x &= 180° - 130° \\ &= 50° \end{aligned} \qquad \text{and} \qquad \begin{aligned} y + 110° &= 180° \\ y &= 180° - 110° \\ &= 70° \end{aligned}$$

d. Recall that the angle between a tangent and a chord is equal to the inscribed angle on the opposite side of the chord. Therefore, $x = 40°$.

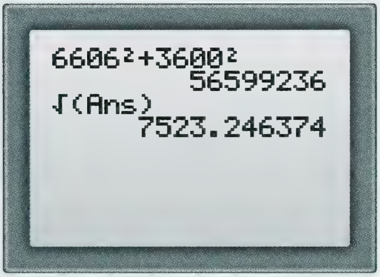
The measure of a central angle is twice the measure of an inscribed angle subtended by the same arc.

$$\begin{aligned} \therefore\ y &= 2x \\ &= 2(40°) \\ &= 80° \end{aligned}$$

8. Draw a diagram showing the information from the problem. Apply the Pythagorean Theorem to find the distance to the centre of Earth.



$$
\begin{aligned}
(SC)^2 &= (AC)^2 + (SA)^2 \\
(SC)^2 &\doteq (6606)^2 + (3600)^2 \\
(SC)^2 &\doteq 56\,599\,236 \\
(SC)^2 &\doteq \sqrt{56\,599\,236} \\
SC &\doteq 7523
\end{aligned}
$$



```
6606²+3600²
           56599236
√(Ans)
        7523.246374
```

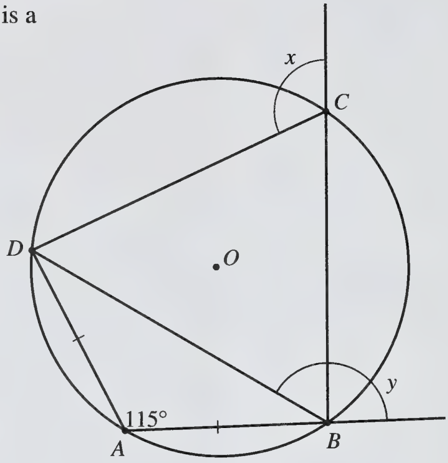
The distance from the satellite to the centre of Earth is approximately 7523 km.

$$
\begin{aligned}
\therefore\ x &\doteq 7523 - 6606 \\
&\doteq 917
\end{aligned}
$$

The distance to the nearest point on Earth from the satellite, *N*, is approximately 917 km.

Follow-up Activities (continued)

11. **a.** Label the diagram. Notice that quadrilateral *ABCD* is a cyclic quadrilateral.



First, find the value of *x*. Notice that *x* is supplementary (angles add up to 180°) to $\angle BCD$ and that $\angle BCD$ is the opposite angle of $\angle BAD$ in a cyclic quadrilateral. So, you need to find the measure of $\angle BCD$ to find the value of *x*.

$$\begin{aligned}\angle BCD &= 180° - \angle BAD\\ &= 180° - 115°\\ &= 65°\end{aligned}$$

$$\begin{aligned}\therefore\ x &= 180° - \angle BCD\\ &= 180° - 65°\\ &= 115°\end{aligned}$$

Now, find the value of *y*.

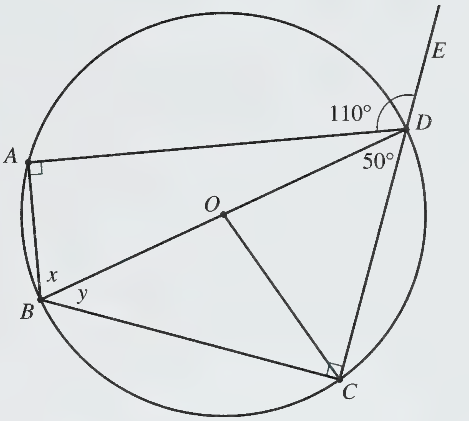
Notice that triangle *ABD* is isosceles; so, $\angle ADB$ and $\angle ABD$ are equal. Also, notice that *y* is supplementary to $\angle ABD$. So, you need to find the measure of $\angle ABD$ first.

The sum of the angles of a triangle is 180°.

$$\begin{aligned}\angle ABD + \angle ADB + 115° &= 180°\\ 2(\angle ABD) + 115° &= 180° \quad \longleftarrow \angle ABD = \angle ADB\\ 2(\angle ABD) &= 180° - 115°\\ 2(\angle ABD) &= 65°\\ \angle ABD &= 32.5\end{aligned}$$

$$\begin{aligned}\therefore\ y &= 180° - \angle ABD\\ &= 180° - 32.5°\\ &= 147.5°\end{aligned}$$

b. Both $\angle BAD$ and $\angle BCD$ are right angles because they are subtended by the diameter BOD.



The sum of the angles of a triangle is 180°.

$$\begin{aligned} \therefore\ \angle CBD + \angle BCD + \angle BDC &= 180° \\ y + 90° + 50° &= 180° \\ y + 140° &= 180° \\ y &= 180° - 140° \\ &= 40° \end{aligned}$$

To find the value of x, you need to find the measure of $\angle ADB$ first.

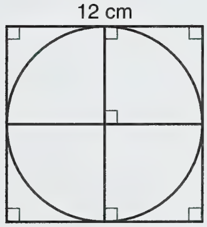
$\angle ADE$, $\angle ADB$, and $\angle BDC$ are supplementary.

$$\begin{aligned} \therefore\ \angle ADE + \angle ADB + \angle BDC &= 180° \\ 110° + \angle ADB + 50° &= 180° \\ \angle ADB + 160 &= 180° \\ \angle ADB &= 180° - 160° \\ &= 20° \end{aligned}$$

The sum of angles of a triangle is 180°.

$$\begin{aligned} \therefore\ \angle BAD + \angle ADB + \angle DBA &= 180° \\ 90° + 20° + x &= 180° \\ x &= 180° - 110° \\ &= 70° \end{aligned}$$

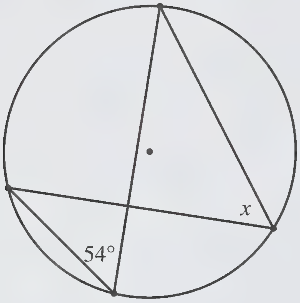
12. Draw a diagram showing the situation given in the question.



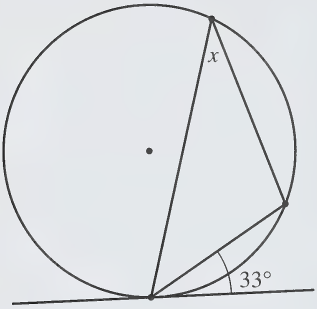
Since the diameter of the circular disk is 12 cm, the length of the side of the box must be 12 cm as well, or the disk would not touch the sides of the box. (This is assuming the sides of the base have no thickness.)

Extra Help

1.

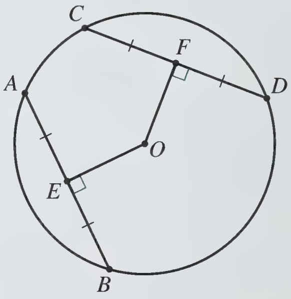


2.



Enrichment

1. **Textbook exercises 1.d., 1.e., and 1.g. of Part A of "What Should I Be Able to Do?," p. 349**



1. d. The line segments are the same length.

e. For different chords having the same length, the line connecting the midpoint of each chord with the centre of the circle will be the same length.

g. Chords of equal length in a circle are equal distances from the centre of the circle.

2. **Textbook exercise 7 of "Exercises: Extending Your Thinking," p. 345**

7. Answers will vary. Use the sample on page 345 as a guide for your creations. You could also make and use a paper template to create your tessellation.

Module Project: Circle Design

Textbook exercises 1 and 2 of Part C of "What Should I Be Able to Do?," pp. 352 and 353

1. The art other than the Yin and Yang symbol appears to be original, which is a requirement, and is quite appealing with good use of colour. The artwork is clean and carefully drawn.

2. There are a few properties of circles used in the drawing. There are more that could have been used in the design though. Where the properties are used is not clearly marked on the diagram, but the properties are shown in the written work. There are several errors in the properties listed in the students' work: the second property should state $\overline{AB} = \overline{AC} = 7.4$ cm, not $\overline{AB} = \overline{AB} = 7.4$ cm; *D* is used twice as a label, thus can lead to confusion; and the word *tangents* is one word, not two.

   The report is clear, easy to understand, and not overly long. There are, however, no examples given and no indication that the work of art constructed was a neon sign. The work shows that research was done, but there are no references given as to the source of the information used, like websites, books, magazines, or interviews with artists.

   This sample project was graded in four areas, each worth 5 marks.

Originality	1	2	3	4	[✓ 5]
Artistic use of the media	1	2	3	[✓ 4]	5
Use of circle properties	1	2	[✓ 3]	4	5
Quality of the report	1	[✓ 2]	3	4	5

   This gives the project a mark of 14 out of 20; therefore, for this module, the student would receive 28 out of a possible 40 marks.

IMAGE CREDITS

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PhotoDisc, Inc.

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